Chapter 8: Comparing Multiple Population Means

One Way Analysis of Variance (ANOVA)

What? Use to compare multiple population or treatment means

How? Works by analyzing this ratio:

$$F = \frac{variation\ between\ groups}{variation\ within\ groups}$$

Idea:

Comparing multiple population means using Analysis of Variance (ANOVA)
Why? ANOVA is a significance test used to compare 2 or more unknown population or treatment means.

When?

1. When comparing population means, the k samples are random and independently selected.
2. When comparing treatment means, subjects are randomly assigned to treatments.
3. The populations or treatment responses are normally distributed.
4. The populations or treatment responses have equal variance (equal standard deviations).

How? ANOVA compares means by analyzing the ratio

$$F = \frac{\text{variation between groups}}{\text{variation within groups}}$$

Here are the steps:

Step 1: Define the k population or treatment means being compared in the context of the problem.
Step 2: Hypotheses: $H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$, H_a = Not all of the population means are equal (No claim as to which means are different.)
Step 3: Choose α.
Step 4: Check the conditions necessary to do the test and see if any necessary assumptions are reasonable.
Step 5: Calculate the test statistic. For ANOVA this is a pretty involved task. First collect all of this information:

$$\begin{aligned}
k &= \text{number of population/treatment means being compared} \\
N &= \text{overall (combined) sample size} \\
n_i &= \text{size of the } i\text{th sample } (N = n_1 + n_2 + \ldots + n_k). \\
\bar{x}_i &= \text{mean of the } i\text{th sample} \\
s_i &= \text{standard deviation of the } i\text{th sample} \\
\bar{x} &= \bar{x} = \text{overall mean (grand mean) } \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \ldots + n_k\bar{x}_k}{N} \\
SSG &= \sum n_i(\bar{x}_i - \bar{x})^2 \text{ (Sum of Squares Groups (or SSTR for Treatments))} \\
SSE &= \sum (n_i - 1)s_i^2 \text{ (Sum of Squares of Errors)} \\
SSTo &= SSG + SSE \text{ (Sum of Squares Total)}
\end{aligned}$$

Then fill in an ANOVA table:

Source of Variation	Degrees of Freedom df	Sum Squares SS	Mean Squares MS	F
Treatments	$k-1$	SSG	$MSG = \frac{SSG}{k-1}$	$F = \frac{MSG}{MSE}$
Errors	$N-k$	SSE	$MSE = \frac{SSE}{N-k}$	
Total	$N-1$	$SSTo$		

Step 6: P-value. The F test statistic comes from an F distribution with DF of Numerator = $k-1$ and DF of Denominator = $N-k$. The larger F is the more evidence against H_0 so the P-value is the probability of getting a larger value of F. On your TI-83, P = fcdf(F,E99,df_{num},df_{den}).
Step 7: Conclusions (as usual!).

Example:

	Sample from population 1	**Sample from population 2**	**Sample from population 3**
n	**6**	**6**	**6**
$\bar{x}$	**52.8**	**67.3**	**63.5**
s	**11.3**	**6.2**	**8.1**

Which means are different?

There are many different kinds of *post hoc* analysis that are possible. "Post hoc" means that we are looking to find differences between pairs of means *after* ANOVA has indicated there is a significant difference somewhere.

Your textbook describes the Bonferonni hypothesis test for comparing pairs of means. It is similar to the pooled sample t-test. We are going to do a confidence interval version of the same procedure.

Bonferonni CI's for differences of means: One particular method of **multiple comparisons** that can be used to assess the statistical significance of the differences between pairs of means is the **Bonferonni method** - it attempts to control the overall probability of false rejections. To have a prescribed confidence for *all* of the intervals at once, we need to increase the confidence for each individual interval.

In this method you construct $b = \binom{k}{2} = \frac{k(k-1)}{2}$ confidence intervals, one for each possible difference of means, $\mu_i - \mu_j$. (Note: $\mu_1 - \mu_2$ counts as the same comparison as $\mu_2 - \mu_1$).
These intervals are essentially like pooled-sample t-CI's, but with a couple of adjustments. First, the Mean Square Error is used in place of the pooled variance. Second, if we want overall confidence level (for the combined intervals) of $1-\alpha$, then the the confidence level of each individual CI is $1 - \frac{\alpha}{\frac{k(k-1)}{2}}$. So to find find the critical value for each CI we will look up the critical value corresponding to a two-tailed probability of $\alpha' = \frac{\alpha}{\frac{k(k-1)}{2}}$.
To form confidence intervals for all $\mu_i - \mu_j$ calculate:

$$(\bar{x}_i - \bar{x}_j) \pm t_{1-\alpha'/2}\sqrt{\text{MSE}(\frac{1}{n_i} + \frac{1}{n_j})}$$

where $t_{1-\alpha'/2}$ is the critical value from Table C4 with $df = N - k$ and the appropriate confidence level.

Find all of the CI's and interpret the results simultaneously. For example: "We are 95% confident that population mean 1 is larger than population mean 2 and there are no significant differences between the other means."

HW 11-2: 1, 3, 7, 9 (if we haven't covered the material on Bonferroni intervals yet, then just do the ANOVA test for each problem. If we have covered the material on Bonferroni intervals, then if you reject the H_0 , you should use Bonferroni intervals to determine which means are different.)

Example:

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