Homework 7

Please show all your work. When possible, write your answers in complete sentences. The easier your solution is to read, the easier it is to give you feedback and points.

1. Find the Laplace transform of $f(t) = (3t+1)u_2(t)$.

2. Find $\mathscr{L}^{-1}\left\{\dfrac{e^{-3s}}{s(s^2+3)}\right\}$.

3. Use the Laplace transform to solve the initial-value problems

   (a) $y'' + 3y' + 2y = u_2(t), \quad y(0) = 0, \quad y'(0) = 1$

   (b) $y' + 2y = f(t), \quad y(0) = 0$ where $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$

4. Consider the initial value problem:

$$y'' + 2y' + (1760^2\pi^2 + 4)y = \delta(t - \pi), \quad y(0) = 0, \quad y'(0) = 0$$

   (a) Use Laplace transforms to solve the IVP.

   (b) Use Mathematica to "play" your solution. For instance, the command below plays a standardized middle A note at 440 Hz for 10 seconds (the sound is very flat because it contains ONLY the pure harmonic, or sine wave):

```
Play[ Sin[ 440 * 2 * Pi * t ],{t,0,10}]
```

   You'll probably have to make use of the UnitStep function to define your solution. For convenience, define a function as your solution, e.g.

```
y[t_] = Sin[ 440 * 2 * Pi * t];
```

   (c) Once you've done (b) and defined a function as your solution, try this:

```
Play[ y[t] + y[t/2] + y[2t],{t,0,10} ]
```

   What do you hear? How do the notes change? Musically, what is the difference iin the notes - do you know?

   (d) If you enjoy this sort of thing, try the command below, it plays the middle A note using a square wave instead of a sine wave:

```
Play[ SquareWave[ 440*t ], {t,0,10} ]
```

   Why does the sound seem so different than when we play a pure sine wave? The basic frequency is the same.

   (e) Hand-in a Mathematica printout showing your sound experiments.

5. Suppose that $F(s) = \mathscr{L}\{f(t)\}$ exists for $s > a \geq 0$. Show that is c if a positive constant, then

$$\mathscr{L}\{f(ct)\} = \frac{1}{c}F\left(\frac{s}{c}\right), \quad s > ca.$$

6. Find the eigenvalues and eigenfunctions for:

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(L) = 0$$