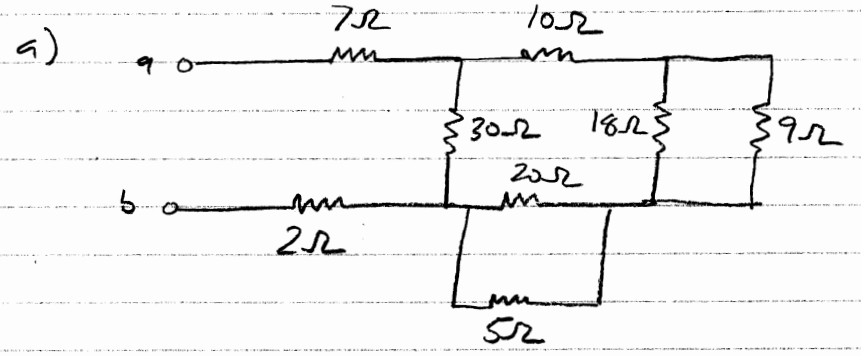
Problem Set 2 (Chapt 2)

(1) P22

Note: // → parallel

a)

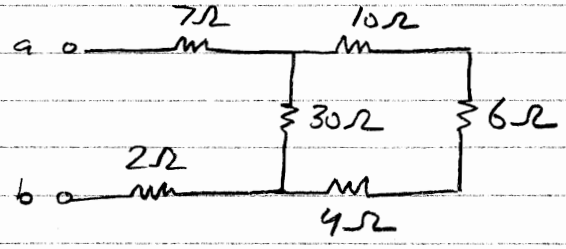


⊙ Determine equivalent resistance R_{eq} between a & b.

- $18\Omega \,//\, 9\Omega = \dfrac{1}{\frac{1}{18\Omega} + \frac{1}{9\Omega}} = \dfrac{1}{\frac{3}{18\Omega}} = 6\Omega$

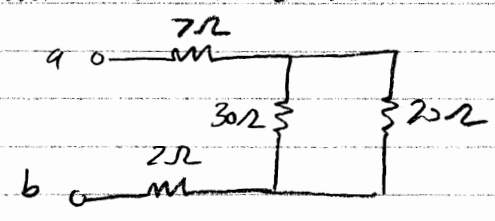
- $20\Omega \,//\, 5\Omega = \dfrac{1}{\frac{1}{20\Omega} + \frac{1}{5\Omega}} = \dfrac{1}{\frac{5}{20\Omega}} = 4\Omega$

We can redraw circuit as



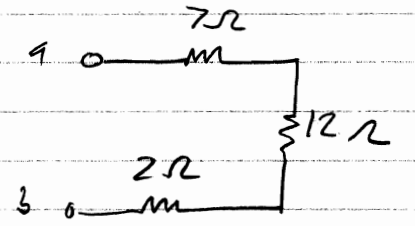
- $10\Omega + 6\Omega + 4\Omega = 20\Omega$

We can redraw circuit as

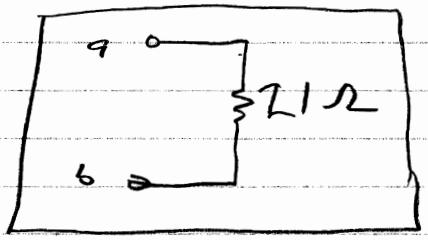


- $30 \,//\, 20\Omega = \dfrac{1}{\frac{1}{30\Omega} + \frac{1}{20\Omega}} = \dfrac{1}{\frac{5}{60\Omega}} = 12\Omega$

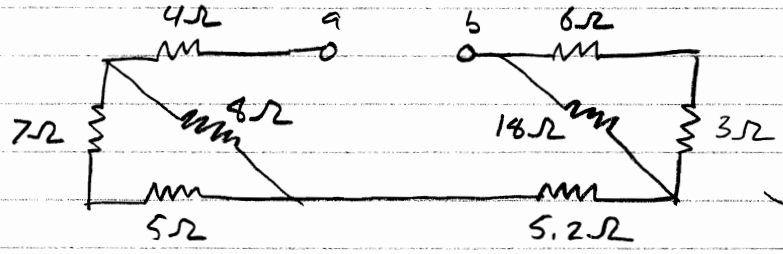
redraw circuit as



- $7\Omega + 12\Omega + 2\Omega = \underline{\underline{21\Omega}}$



b)



this side reduces to:

$$4\Omega + [(7\Omega + 5\Omega) \,||\, 8\Omega]$$

$$= 4\Omega + \left[\frac{1}{\frac{1}{12\Omega} + \frac{1}{8\Omega}}\right]$$

$$= 4\Omega + \frac{24}{5}\Omega = \underline{\underline{\frac{44}{5}\Omega}}$$

this side reduces to:

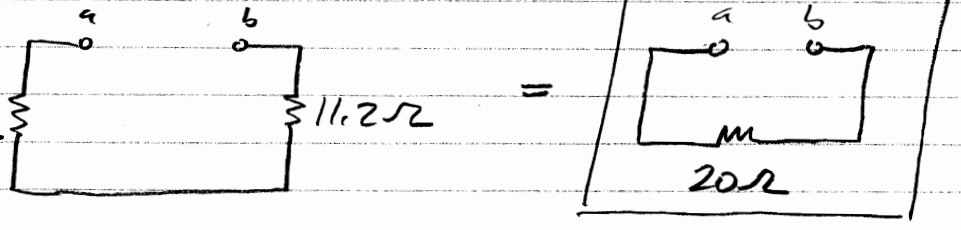
$$\left(18\Omega \,||\, (6\Omega + 3\Omega)\right) + 5.2\Omega$$

$$\left(\frac{1}{\frac{1}{18\Omega} + \frac{1}{9\Omega}}\right) + 5.2\Omega$$

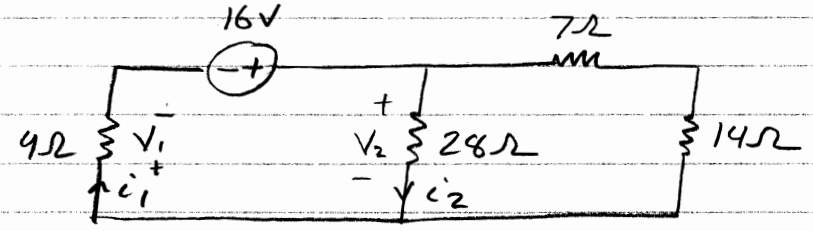
$$6\Omega + 5.2\Omega$$

$$= \underline{\underline{11.2\Omega}}$$

We can redraw the circuit as.



(2.) P2.22 Find the values of i_1 & i_2



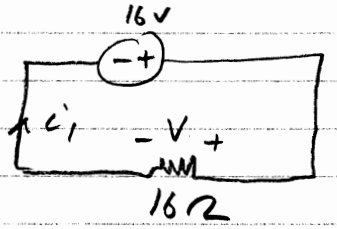
First will find i_1 by drawing equivalent circuit.

$$R_{eq} = [28\Omega \,//\, (7\Omega + 14\Omega)] + 4\Omega$$

$$= \frac{1}{\frac{1}{28\Omega} + \frac{1}{21\Omega}} + 4\Omega$$

$$= 12 + 4\Omega = 16\Omega$$

So equivalent circuit is



KVL: $-16V + V = 0$

$V = 16V$ ✓

Ohm's: $\boxed{i_1 = \frac{V}{R} = \frac{16V}{16\Omega} = 1\,Amp.}$

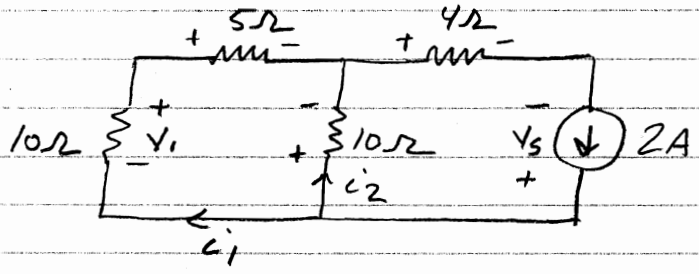
To get i_2, write KVL equation around inner loop.

$$-16V + V_2 + V_1 = 0$$

$$-16V + i_2(28\Omega) + (1A)(4\Omega) = 0$$

$$\boxed{i_2 = \frac{16V - 4V}{28\Omega} = 0.43\,Amps} \checkmark$$

(3.) P 2.23 Find the values of V_s, V_1, & i_2



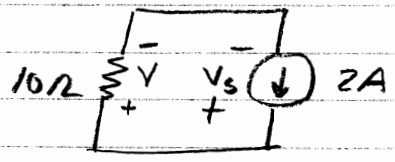
First, will get V_s by combining R's into equivalent resistance.

$$R_{eq} = 4\Omega + \left[10\Omega \,//\, (10\Omega + 5\Omega)\right]$$

$$= 4\Omega + \left[\frac{1}{\frac{1}{10\Omega} + \frac{1}{15\Omega}}\right]$$

$$= 4\Omega + 6\Omega = \underline{\underline{10\Omega}}.$$

∴ Equavalut circuit is



KVL: $-V_s + V = 0$

$$\boxed{V_s = V = (2A)(10\Omega) = 20V} \checkmark$$

Now can find i_1 & i_2 by writing KVL equations around loops of original circuit.

$$-V_s + (10\Omega)(i_2) + (4\Omega)(2A) = 0$$

$$\boxed{i_2 = \frac{V_s - 8V}{10\Omega} = \frac{20V - 8V}{10\Omega} = 1.2\,Amps} \checkmark$$

likewise:

$$-V_s - V_1 + (5\Omega)(i_1) + (4\Omega)(2A) = 0$$

$$V_1 = 8V - V_s + (5\Omega)(i_1)$$

what is i_1?

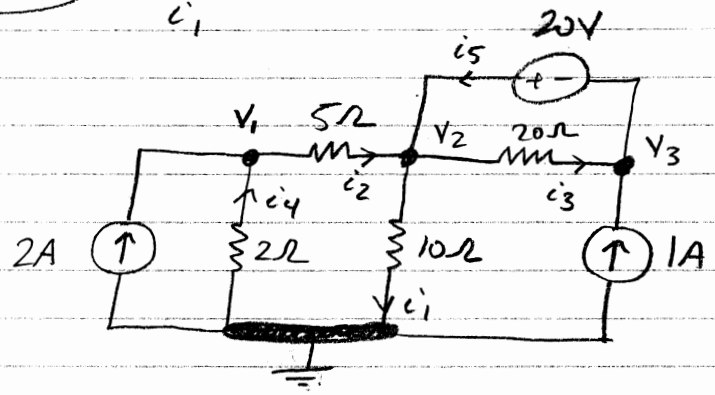
KCL: $i_1 + i_2 = 2A$

$$i_1 = 2A - i_2 = 2A - 1.2A = 0.8A. \checkmark$$

$\therefore V_1 = 8V - 20V + (5\Omega)(0.8A)$

$$\boxed{V_1 = -8V} \checkmark$$

(4) P2.43 Use node-voltage technique to find i_1



1. ID nodes.
2. Assign currents
3. Assign voltages & choose ground.
4. Obvious relations:

$$V_2 - V_3 = 20V$$

5. Apply KCL

V_1: $2A + i_4 = i_2$

V_2: $i_2 + i_5 = i_1 + i_3$ } supernode: $i_1 + i_3 - i_2 = i_3 + 1A$

V_3: $i_3 + 1A = i_5$ $\quad i_1 - i_2 = 1A$

ground: $2A + 1A + i_4 = i_1$

6. Write i's in terms of v's

$$V_1: \quad 2A + \frac{0 - V_1}{2\Omega} = \frac{V_1 - V_2}{5\Omega}$$

$$\text{Supernode:} \quad \frac{V_2 - 0}{10\Omega} - \frac{V_1 - V_2}{5\Omega} = 1A$$

$$\text{ground:} \quad 3A + \frac{0 - V_1}{2\Omega} = \frac{V_2 - 0}{10\Omega}$$

} Have 3 eqns and 2 unknowns

Solvable ✓

7. Solve system:

V_1: $(2A)(10\Omega) - 5V_1 = 2(V_1 - V_2)$

$$20V - 7V_1 + 2V_2 = 0$$ ✓

supernode:

$$V_2 - 2V_1 + 2V_2 = 10V$$

$$V_2 = \frac{10V + 2V_1}{3}$$ ✓ ← sub into V_1 eqn.

$$20V - 7V_1 + 2\left(\frac{10V + 2V_1}{3}\right) = 0$$

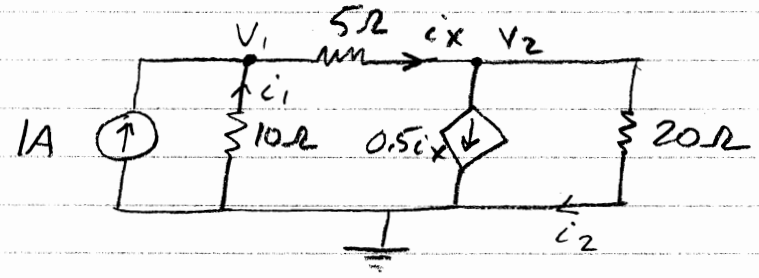
$V_1 = 4.7\ V$ ✓

$V_2 = 6.5\ V$

8. Use V_2 to solve for i_1.

$$i_1 = \frac{V_2 - 0}{10\Omega} = \frac{6.5\ V}{10\Omega} = \boxed{0.65A}$$ ✓

(5) P2.46. Solve for values of node voltages V_1 & V_2, then solve for i_x.



1. ID nodes
2. Assign currents
3. Assign voltages
4. Notice obvious relations - none that I can see
5. KCL

V_1: $1A + i_1 = i_x$

V_2: $i_x = 0.5i_x + i_2$

$0.5i_x = i_2$

ground: $i_2 + 0.5i_x = i_1 + 1A$

6. Write i's in terms of V's

V_1: $1A + \frac{0 - V_1}{10\Omega} = \frac{V_1 - V_2}{5\Omega}$

V_2: $0.5\left(\frac{V_1 - V_2}{5\Omega}\right) = \frac{V_2 - 0}{20\Omega}$

ground: $\frac{V_2 - 0}{20\Omega} + 0.5\left(\frac{V_1 - V_2}{5\Omega}\right) = \frac{0 - V_1}{10\Omega} + 1A$

7. Solve system:

V_1: $(10\Omega)(1A) - V_1 = 2V_1 - 2V_2$

$10V - 3V_1 + 2V_2 = 0$

V_2 : $2V_1 - 2V_2 = V_2$

$2V_1 = 3V_2$

$\therefore V_2 = \frac{2}{3} V_1$ ← plug into eq for V_1

$10V - 3V_1 + 2\left(\frac{2}{3} V_1\right) = 0$

$10V - 1.67 V_1 = 0$

$\boxed{V_1 = 6 \ldots V}$

$V_2 = \frac{2}{3}(6V) = \boxed{4V}$

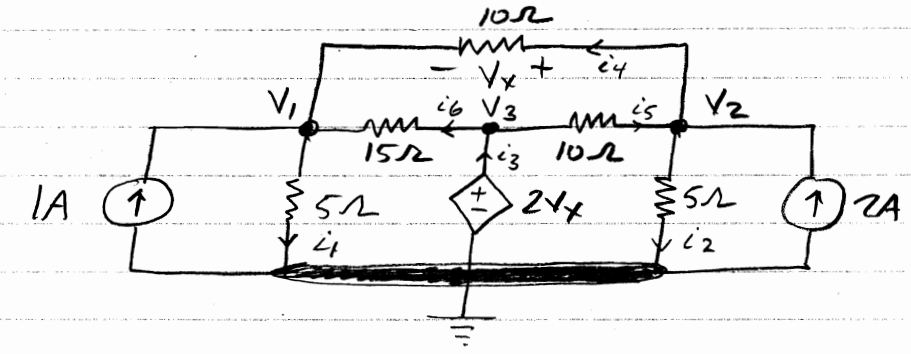
4. Solve for i_x

$i_x = \frac{V_1 - V_2}{5\Omega} = \frac{6V - 4V}{5\Omega} = \boxed{0.4 \text{ Amp.}}$ ✓

■

P 2.48

(6) Solve for node voltages V_1 & V_2, then find V_x.



1. ID nodes
2. Assign currents
3. Assign voltages to nodes
4. obvious relations:

$V_3 - 0 = 2V_x$ $\qquad$ $V_x = V_2 - V_1$

$V_3 = 2V_x$

5. Apply KCL

V_1: $1A + i_6 + i_4 = i_1$

V_2: $2A + i_5 = i_2 + i_4$

V_3: $i_3 = i_6 + i_5$

ground: $1A + 2A + i_3 = i_1 + i_2$

} Super node:

$i_6 + i_5 = i_1 + i_2 - 3A$

6. i's $\rightarrow$ v's

V_1: $1A + \frac{V_3 - V_1}{15\Omega} + \frac{V_2 - V_1}{10\Omega} = \frac{V_1}{5\Omega}$

$30V + 2V_3 - 2V_1 + 3V_2 - 3V_1 = 6V_1$

$30V - 11V_1 + 3V_2 + 2V_3 = 0$

V_2: $2A + \frac{V_3 - V_2}{10\Omega} = \frac{V_2}{5\Omega} + \frac{V_2 - V_1}{10\Omega}$

$20V + V_3 - V_2 = 2V_2 + V_2 - V_1$

$\underline{20V + V_1 - 4V_2 + V_3 = 0}$

Supernode: note: the supernode current equation is a superposition of V_1 & V_2 equations, therefore does not provide additional information.

Other equations we can use are: $V_3 = 2V_x$ & $V_x = V_2 - V_1$

$\therefore \underline{V_3 = 2V_2 - 2V_1}$

7. Solve system:

$V_3 = 2V_2 - 2V_1$

plug into other two eqns:

- $20V + V_1 - 4V_2 + 2V_2 - 2V_1 = 0$

  $\underline{20V - V_1 - 2V_2 = 0}$

- $30V - 11V_1 + 3V_2 + 2(2V_2 - 2V_1) = 0$

  $\underline{30V - 15V_1 + 7V_2 = 0}$

$-15(20V - V_1 - 2V_2) = 0$

$+ \quad 30V - 15V_1 + 7V_2 = 0$

$-270V + 37V_2 = 0$

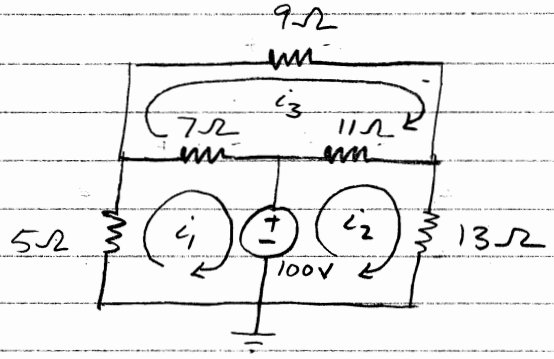
$\therefore \boxed{V_2 = 7.3V}$ ✓

now solve for V_1

$$V_1 = \frac{7(V_2) + 30V}{15} = \frac{7(7.3) + 30V}{15}$$

$$\boxed{V_1 = 5.4\,V} \checkmark$$

(7) P 2.53



Solve for mesh currents shown (i_1 i_2 i_3)

1. 1D loops
2. Assign currents
3. Obvious relations: none
4. Apply KVL

i_1: $100V + V_{5\Omega} + V_{7\Omega} = 0$

i_2: $-100V + V_{11\Omega} + V_{13\Omega} = 0$

i_3: $V_{9\Omega} + V_{11\Omega} + V_{7\Omega} = 0$

5. $V_i \rightarrow i$'s

i_1: $100V + i_1\,5\Omega + (i_1 - i_3)\,7\Omega = 0$
$100V + i_1\,12\Omega - i_3\,7\Omega = 0$

i_2: $-100V + (i_2 - i_3)\,11\Omega + i_2\,13\Omega = 0$
$-100V + i_2(24\Omega) - i_3\,11\Omega = 0$

i_3: $i_3\,9\Omega + (i_3 - i_2)\,11\Omega + (i_3 - i_1)\,7\Omega = 0$
$i_3\,27\Omega - i_2\,11\Omega - i_1\,7\Omega = 0$

6. Solve system

- from i_1: $\quad i_1 = \dfrac{i_3\,7\Omega - 100V}{12\Omega}$

$$i_1 = \frac{7}{12} i_3 - 8.33A$$

- plug into eq for i_3:

$$i_3\,27\Omega - i_2\,11\Omega - 7\Omega\left(\frac{7}{12} i_3 - 8.33A\right) = 0$$

$$i_3\,27\Omega - i_2\,11\Omega - 4.1\Omega\, i_3 + 58.3V = 0$$

$$i_3\,22.9\Omega = i_2\,11\Omega - 58.3V$$

$$i_2 = \frac{i_3\,22.9\Omega + 58.3V}{11\Omega} = 2.08\, i_3 + 5.3A$$

- plug into eq for i_2:

$$-100V + (2.08\, i_3 + 5.3A)\,24\Omega - i_3\,11\Omega = 0$$

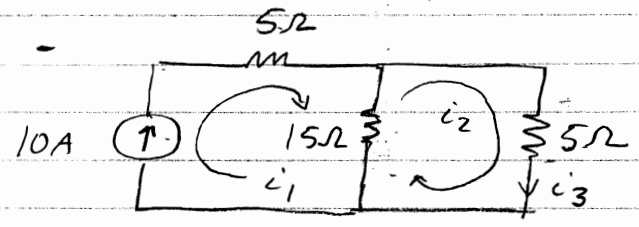
$$-100V + 49.9\, i_3 + 127.2V - i_3\,11\Omega = 0$$

$$\boxed{i_3 = -0.7A.} \checkmark$$

$$i_2 = 5.3A + 2.08(-0.7A) = \boxed{3.8A} \checkmark$$

$$i_1 = \frac{7}{12}(-0.7A) - 8.33A = \boxed{-8.7A} \checkmark$$

(8) P 2.60



Use Mesh analysis to obtain i_3

1. 10 loops
2. Assign currents
3. Obvious relations: $i_2 = i_3$, $i_1 = 10A$
4. KVL.

i_1 : not a good one

i_2 : $V_{5\Omega} + V_{15\Omega} = 0$

5. Vs → is

i_1 : not a good one

i_2 : $i_2 5\Omega + (i_2 - i_1) 15\Omega = 0$

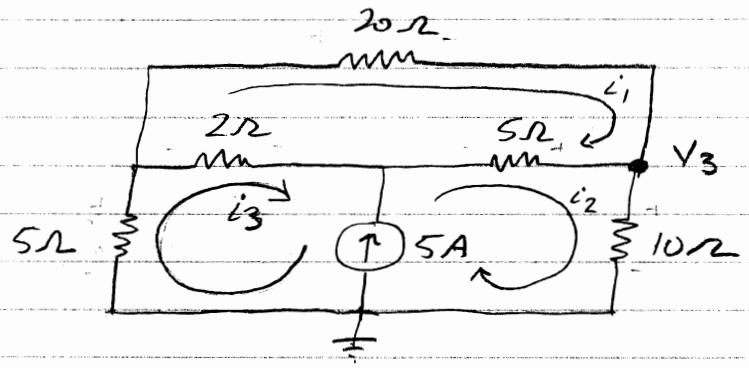
$$i_2 20\Omega - i_1 15\Omega = 0$$

6. Solve system

$$i_2 20\Omega - (10A) 15\Omega = 0$$

$$i_2 = \boxed{7.5 \text{ Amps} = i_3} \checkmark$$

(9.) P 2.62 Use mesh analysis to find V_3



1. 10 loops
2. Assign currents
3. obvious relations $i_2 - i_3 = 5A$ ①
4. KVL:

i_1: $V_{20\Omega} + V_{5\Omega} + V_{2\Omega} = 0$

i_2: not a good one

i_3: not a good one

5. $V_s \rightarrow i's$

$$i_1: \; i_1\,20\Omega + (i_1 - i_2)5\Omega + (i_1 - i_3)2\Omega = 0$$
$$i_1\,27\Omega - i_2\,5\Omega - i_3\,2\Omega = 0 \;②$$

note: we have 3 unknowns but only two eqns.
-need one more.

Write KVL eqn around loop containing both i_3 & i_2

$$i_2\,10\Omega + i_3\,5\Omega + (i_3 - i_1)2\Omega + (i_2 - i_1)5\Omega = 0$$
$$-i_1\,7\Omega + i_2\,15\Omega + i_3\,7\Omega = 0 \;③$$

6. Solve system

from ① get $\underline{i_2 = 5A + i_3}$

- plug this into eq ② & ③

$i_1\,27\Omega - (5A + i_3)\,5\Omega - i_3\,2\Omega = 0$

$\underline{i_1\,27\Omega - i_3\,7\Omega - 25V = 0}$

- $-i_1\,7\Omega + (5A + i_3)\,5\Omega + i_3\,7\Omega = 0$

$\underline{-i_1\,7\Omega + i_3\,22\Omega + 75V = 0}$

$i_1 = \dfrac{+75V + i_3\,22\Omega}{7\Omega}$

- $\left(\dfrac{75V + i_3\,22\Omega}{7\Omega}\right) 27\Omega - i_3\,7\Omega - 25V = 0$

$289.3V + 84.9\Omega\, i_3 + i_3\,7\Omega - 25V = 0$

$i_3 = -\dfrac{264.3V}{77.9\Omega} = -3.4\ \text{Amps}$

7. Use i_3 to get i_2 and V_3

$i_2 = 5A + i_3 = 5A - 3.4\ \text{Amps} = 1.6\ \text{Amps}$

$V_3 - 0 = i_2\,10\Omega = (1.6A)\,10\Omega = \boxed{16V}$