Homework:

Sec. 11.2 (last time): # 1, 3, 9, 19, 25, 27, 31-34

Now add: 11.2 #36, 37, 41, 42*, 51-56, 57-58.
(* interpret your answer -- is that alot?),

Present *Monday*: 11.2 #37, 42, 54

9:55 - Mae Br., Aspyn, Abby, Kelly, Timothy, Cory Cu.
11:00 - Courtney Ha., Sarah, Tyler, Melissa, Samantha, Courtney Ho.

See handout for your assignment for Wednesday and Thursday of this week.

I am available for Office Hours on Wednesday -- 9am to 10am only! (my plane leaves at 11:30am).

Brief Interlude -- Scientific Notation

Some of the numbers we deal with in counting problems (and the related probability problems) are either very large or very small.

For instance:

The probability that any two randomly selected individuals have the same birthday is approximately 0.0027. $= 2.7 \times 10^{-3}$ $= 2.7E{-3}$

(Curiously, the probability that any group of 37 randomly selected individuals will have at least one "birthday pair" is 0.847!) $= 8.47 \times 10^{-1}$

The number of ways a deck of 52 cards can be arranged is 52!. See next page for more details on that whopper....

Factorial notation:

For any whole number n, the notation $n!$ means the product $(n)(n-1)(n-2)\ldots(1)$.
(We also define $0! = 1$).

For instance, $6! = (6)(5)(4)(3)(2)(1) = 720$.

Note: Factorials grow quickly! See below...

20! = 2,432,902,008,176,640,000 ≈ 2.4 x 10^{18}
(over two billion billion)

52! = 80,658,175,170,943,878,571,660,636,856,403,766,975,289,505,440,883,277,824,000,000,000,000
(about 8.1 x 10^{68}, which is an *astronomically* large number... there are only about 3 x 10^{23} stars in the visible universe)

Factorial Examples: Calculate by hand (simplify first where possible):

A) $(7 - 2)! =$ $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
(Contrast with 7! - 2!)

B) $5! \times 3! =$ $120 \times 6 = 720$
(Contrast with 15!)

Factorial Examples: Calculate by hand (simplify first where possible):

C) 6! / 3! = $\frac{6\cdot 5\cdot 4\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 120$

(contrast with 2!)

D) (7! x 2!) / (5!) = $\frac{(7\cdot 6\cdot \cancel{5!}) \times 2\cdot 1}{(\cancel{5!})} = 84$

Ex: $8! = 8\cdot 7!$

More Examples: Here are the names of the members of the Teacher Education Club. Freshmen are noted with (*).

Members:
{Amy*, Beth, Craig*, Danielle*, Ethan, Francine, George}

1. In how many ways can they be lined up for a photograph?

$7! = 5{,}040$

2. In how many ways can the Teacher Education Club elect a president and vice-president from its members?

choice(s)	pres.	vice. pres.	total
# ways	7	6	$7\cdot 6 = 42$

More Examples: Here are the names of the members of the Teacher Education Club. Freshmen are noted with (*).

Members:
{Amy*, Beth, Craig*, Danielle*, Ethan, Francine, George}

3. How many ways can a president and vice-president be elected if the vice-president must be a freshman?

choices	pres.	V.P.	total
# ways	7	~~3~~ 2 or 3 (it depends)	~~21~~?

With this sequence of choices, the uniformity criterion is not met.

choices	V.P.	pres.	total
# ways	3	6	18 ways

4. How many ways can a pair of members be selected to travel to the regional conference in Minneapolis?

	A	B	C	D	E	F	G
A	~~AA~~	AB	AC	AD	AE	AF	AG
B	BA	~~BB~~					
C							
D							
E							
F							
G							

21 ways

... etc ...

Duplicates

More challenging example: *see #52-55 for similar problems

The members of the club are to be seated at an awards ceremony. Seven seats have been reserved for the members.

Club = {Amy*, Beth, Craig*, Danielle*, Ethan, Francine, George}

If Amy and Beth insist on being seated next to one another, how many ways are there to seat all 7 members?

Hint: Break the task into the following choices, and count how many ways there are to make each choice in turn:

6 ← 1. Pick a pair of seats the pair AmyBeth.

2 ← 2. Count ways to seat AmyBeth in their adjacent seats.

5! 3. Assign the rest of the seats.

_ _ _ _ X X _
1 2 3 4 5 6 7

choice	1. (above)	2. (above)	3. (above)	Total
# ways	6	2	5!	1440 ways

Self Check

How many 3-digit numbers can be formed using the digits in the set {1, 2, 3, 4, 5} if...

i) ...you cannot use any digit more than once? $5 \cdot 4 \cdot 3 = 60$ ways

ii) ...you can use digits as many times as you like? $5 \cdot 5 \cdot 5 = 125$ ways

STOP and check your answers with your neighbor. Then work together on the following:

iii) ...you must use one number twice (with no other repeats).

Think carefully about the choices.

5 - choose the pair

3 - place the pair → e.g. X _ X

4 - place the last digit.

60 ways