

### Distributions and Rank

**Present Today:** #2, 3a, and 5

9:55 Timothy Chr., Cory, Kaitlin, Amber, Alyssa, Molly Gio.

11:00 Sam Hod., Courtney, Jennifer, Taylor, Elizabeth, Kamry Lon.

Discuss other answers to worksheet.

Homework: 13.3 and 13.4:

13.3 29-32, 35-37

13.4 #1, 3, 5, 7, 21, 34

Present: 13.3 #37, and 13.4 #1 and 21

9:55 Kaitlin Hei., Kaitlyn, Mandi, Grace, Molly, Azjja Kjo.

11:00 Caitlyn McL., Gen, Danielle, Steven, Emily, Jordan Pet.

$$\begin{array}{r} 3b) \quad \text{GPA} \quad \text{Credits} \\ \quad \quad 2.63 \quad 56 \\ \quad \quad 3.115 \quad 13 \\ \hline \quad \quad \frac{(2.63)(56) + (3.115)(13)}{56 + 13} \end{array}$$

So far we've **summarized data** (frequencies, distributions, center, and spread) and we've **compared samples** (left- and right-hand reaction times).

Today, we look at ways to comparing individuals relative to a sample or population.

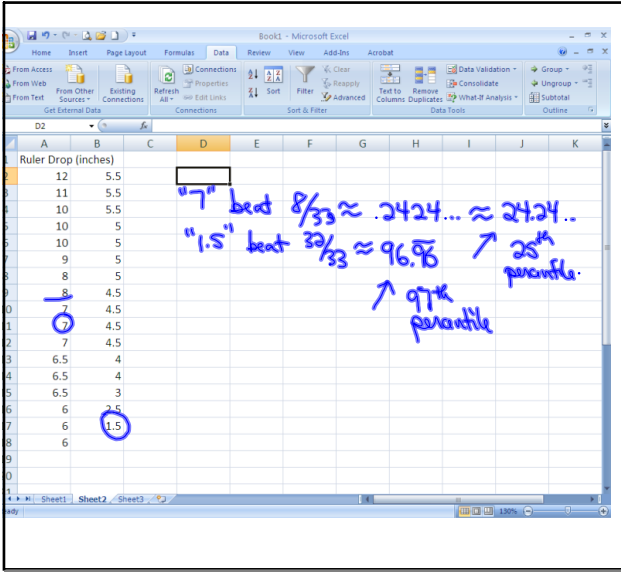
Let's first generate some *individual* and *group* data. We'll do the ruler drop experiment again, this time each student contributes *one* data point.



### Measures of Position: Percentiles, Quartiles, Deciles

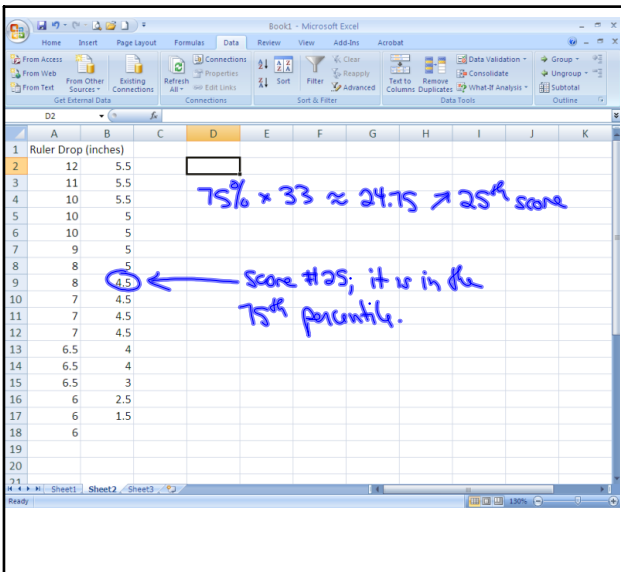
A data point  $x$  is said to be in the  **$n$ th percentile** if approximately  $n\%$  of the data points are less than  $x$ . So percentile rank ranges from the 1st percentile to the 99th percentile.

Calculate your percentile rank for the ruler drop dataset we just collected.



Measures of Position:  
Percentiles, Quartiles, Deciles

Follow-up:  
 Who here is in the 75th percentile? (faster than 75% of all reaction times today?)



Measures of Position:  
Percentiles, Quartiles, Deciles

The **decile rank** for a data point marks which 10th percentile it is in. (Note that the 25th percentile is said to be in the 3rd decile...)

%-ile	0-9	10-19	20-29	30-39	40-49	50-59	...	90-99
decile	1st	2nd	3rd	4th	5th	6th		10th

Handwritten red notes:

- quartiles: 25%, 25%

What is your **decile rank** for the reaction time data?

Measures of Position:  
Percentiles, Quartiles, Deciles

Similarly, the **quartile rank** for a data point marks which **quarter** of the data it is in.

**Example:** Find the numbers that divide the reaction time data set into quarters. **These three numbers are the upper limits of (and are commonly referred to as) the 1st, 2nd, and 3rd quartiles.**

Note that the 2nd quartile is just the **median**.

Using Rank to Compare Performances

Here's an example from the (now outdated) WKCE report. How did this student do in each of the four subject areas tested relative to other students in the state?

**Scale Scores and State Percentiles**

	Score Range	Cut-Scores			Your Student	State Percentile
		Basic	Proficient	Advanced		
Reading	280-650	396	440	489		
→ Language Arts	140-420	252	277	308	385	93
→ Mathematics	240-650	421	438	484	430	42
Science	170-440	249	279	320	329	77
Social Studies	170-400	242	263	288	271	58

<http://dpi.wi.gov/oea/pdf/adminguide.pdf>

<http://dpi.wi.gov/oea/publications.html>

Measures of Position:  
Standardized Scores

Another important measure of position (or rank) is the **z-score** or **standardized score**. It expresses the score in terms of its distance (in standard deviations) from the center.

**Example:** Three cousins, Alice, Becky, and Chuck, took a history test at their school, but all three students attend different schools. Here are the results:

	Score	Mean	Std. Dev.	
2 Alice:	78	75	6	$z = \frac{78-75}{6} = +\frac{1}{2}$
1 Becky:	71	68	3	$z = \frac{71-68}{3} = +1$
3 Chuck:	80	82	4	$z = \frac{80-82}{4} = -\frac{1}{2}$

Rank the cousins in terms of who did the best, relative to the others in their class.

Measures of Position:  
Standardized Scores

**Definition:** The **z-score** (a measure of position) for a given data point  $x$  is defined as:

$$z = \frac{x - \bar{x}}{s}$$

where  $\bar{x}$  represents the mean and  $s$  represents the standard deviation for the data set.

Calculate the missing piece:

A) Math test:  $x = 82$ , mean = 70,  $s = 6$ ,  $z = +2$

$$\frac{82-70}{6} = \frac{12}{6} = 2$$

Measures of Position:  
Standardized Scores



B) Price of gas:  $x = 1.83$ , mean = \$1.95,  $s = 0.06$ ,  $z = -2$

$$z = \frac{x - \bar{x}}{s}$$

$$-2 = \frac{x - 1.95}{0.06} \Rightarrow \text{solve for } x \quad (x = 1.83)$$

C) Miles per month:  $x = 480$ , mean = \_\_\_\_\_,  $s = 80$ ,  $z = -1.5$

$$z = \frac{x - \bar{x}}{s}$$

$$-1.5 = \frac{480 - \bar{x}}{80}$$

$$\Rightarrow -120 = 480 - \bar{x}$$

$$\Rightarrow 600 = \bar{x}$$