

Present Tuesday: #21, 35, 36

9:55 Abby Bro., Kelly, Timothy, Cory, Kaitlin, Amber Eid.
11:00 Tyler Hec., Melissa, Sam, Courtney, Jennifer, Taylor Kru.

Homework: 7.4 #11, 15, 19, 23, 27, 29, 35, 39, 45, 47,
57, 59, 61, 64

Present Wednesday: #27, 47, 61

9:55 Alyssa Fro., Molly, Kaitlin, Kaitlyn, Mandi, Grace Jar.
11:00 Elizabeth Lin., Kamry, Caitlyn, Gen, Danielle, Steve Mil.

After reviewing the correct solution (below), write your score on the back of your quiz.

- 0 = no progress at all; just rewrote problem
- 0.5 = false start, not based on relevant principles
- 1 = false start, but sustained effort with some relevant principles
- 1.5 = significant mistake(s), or significant misunderstanding(s)
- 2 = mistake near the end or could not finish; also excessive reliance on calculator or 'brute force' methods
- 2.5 = trivial mistake (e.g. arithmetic error), but work is mostly correct
- 3 = correct answer and work

A truck radiator holds 18 liters of fluid. How much pure (100%) antifreeze must be added to a mixture that is 4% antifreeze in order to fill the radiator with a mixture that is 20% antifreeze?

Let x = # liters of pure antifreeze.
Let y = # liters of 4% mixture of antifreeze.

Then $x + y = 18$, and $\textcircled{1}$
 $100\%x + 4\%y = 20\%(18)$ $\textcircled{2}$
is the system that solves this problem.

$\textcircled{1} \rightarrow x + y = 18 \Rightarrow x = 18 - y$
 $\textcircled{2} \rightarrow x + .04y = 3.6$
 $\Rightarrow (18 - y) + .04y = 3.6$
 $\rightarrow -96y = -14.4$
 $\Rightarrow y = 15 \text{ liters.}$
 $\Rightarrow x = 3 \text{ liters.}$

Final Example (from the purplemath.com walkthrough):

A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170 graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators must be shipped each day.

If each scientific calculator sold results in a \$2 loss, but each graphing calculator produces a \$5 profit, how many of each type should be made daily to maximize net profits?

x : number of scientific calculators produced
 y : number of graphing calculators produced
 P : profit if x scientific and y graphing calculators are sold

Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day.

$$x \geq 100, \quad y \geq 80$$

Because of limitations on production capacity, no more than 200 scientific and 170 graphing calculators can be made daily.

$$x \leq 200, \quad y \leq 170$$

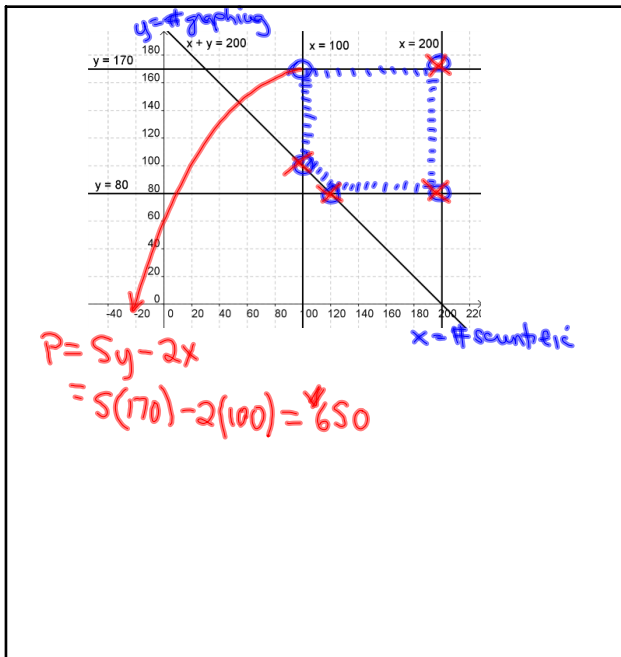
A total of at least 200 calculators must be shipped each day.

$$x + y \geq 200$$

Each scientific calculator sold results in a \$2 loss, but each graphing calculator produces a \$5 profit

$$P = (-2)x + 5(y)$$

$$P = 5y - 2x$$



Some Preliminaries: Sec. 7.4 - Linear Inequalities

Definition: $a < b$ means "a is less than b"
 Definition: $a \geq b$ means "a is either greater than b or equal to b"

Multiplication Property for Inequalities:
 If $c > 0$, then $a < b$ is equivalent to $ac < bc$,
 but if $c < 0$, then $a < b$ is equivalent to $ac > bc$.

Solve: $-3x - 12 \geq 6 - x$ (where is the multiplication property used?)

$$\begin{aligned} -3x - 12 &\geq 6 - x \\ +x & \quad +x \\ -2x - 12 &\geq 6 \\ +12 & \quad +12 \\ -2x &\geq 18 \\ +2x & \quad +2x \\ 0 &\geq 2x + 18 \\ -18 & \quad -18 \\ -18 &\geq 2x \\ -9 &\geq x \\ x &\leq -9 \end{aligned}$$

Notation: The *compound inequality* " $a < x < b$ " is shorthand for " $a < x$ and $x < b$ "
 (Note that by transitivity, it follows that " $a < b$ " as well.)

Caution: There is NO accepted shorthand for " $a < x$ or $x > b$ "!!

The solution to a compound inequality is an interval.

Example: Solve, and justify key steps:
 $1 < \frac{1}{2}(6 - x) < 4$

$$\frac{1}{2}(6 - x) = \frac{6 - x}{2}$$

$$\frac{2}{2} < \frac{6 - x}{2} < \frac{8}{2}$$

checking
 $(-4 < -x < 2) \div -1 \rightarrow -4 < -x \text{ and } -x < 2$
 $4 > x \text{ and } x > -2$
 $4 > x > -2$
 $-2 < x < 4$
 $(-2, 4)$

Conjunction Example -- Find the Error!

Here is Gianni's work for solving a compound inequality. Can you find his error? What is the correct solution?

$$5 \leq 3 - 4x \leq 11$$

$$\Rightarrow 2 \leq -4x \leq 8 \rightarrow -\frac{1}{2} \geq x \geq -2$$

$$\Rightarrow -1/2 \leq x \leq -2 \rightarrow -2 \leq x \leq -1/2$$

Absolute Values and Inequalities:

Recall that $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Which values of x make the following true?

$$|x| < 3 \Rightarrow$$

$$|x| > 5 \Rightarrow$$

$$|x - 3| < 2 \Rightarrow$$

$$|2x + 1| > 2 \Rightarrow$$

A) Mistake? Here's how Charles solved $|2x - 3| < 5$. Can you find his mistake? What is the correct solution?

$$\begin{aligned} &|2x + 3| < 5 \\ \Rightarrow &|2x| < 2 \\ \Rightarrow &|x| < 1 \\ \Rightarrow &-4 < x < 4 \end{aligned}$$

B) Mistake? Here's how Katrina solved $|4 - x| > 3$. Can you find her mistake? What is the correct solution?

$$\begin{aligned} &|4 - x| > 3 \\ \Rightarrow &4 - x > 3 \quad \text{or} \quad 4 - x < -3 \\ \Rightarrow &-x > -1 \quad \text{or} \quad -x < -7 \\ \Rightarrow &x < 1 \quad \text{or} \quad x > 7 \\ \Rightarrow &1 > x > 7 \end{aligned}$$