

Present Thursday: #35, 52, 91

9:55 Molly Kas., Azjja, Katie, Megan, Katie, Alyssa Nei.  
 11:00 Emily New., Jordan, Kyle, Rebecca, Emma, Jamie Sag.

Homework:

Watch your email later today!

Present Monday: Watch your email later today!

9:55 Nicole Nie., Rachel, Hannah, Chelsea, Molly, Amy Sur.  
 11:00 Erin San., Jamie, Victoria, Abbey, Kelly, Jeffrey Ste.

Find the value of  $(x,y)$  that maximizes the equation  $P = 10x + 3y$  subject to the constraints below.

①  $-2(2x+3y) \leq (6) \rightarrow x \geq 0$   
 ②  $4x + y \leq 6 \quad y \geq 0$

Find intersection:

①+②  $\Rightarrow -5y = -6$   
 $\Rightarrow y = \frac{6}{5} = 1.2$

②  $\Rightarrow 4x + 1.2 = 6$   
 $\Rightarrow 4x = 4.8$   
 $\Rightarrow x = 1.2$

So  $(x,y) = (1.2, 1.2)$ .

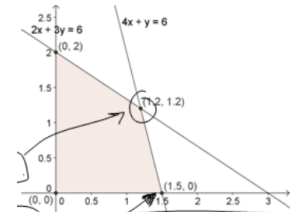
Test vertex points:

$(0,2) \rightarrow P = 10(0) + 3(2) = 6$

$(1.2, 1.2) \rightarrow P = 10(1.2) + 3(1.2) = 15.6$

$(1.5, 0) \rightarrow P = 10(1.5) + 3(0) = 15$

$(0,0) \rightarrow P = 10(0) + 3(0) = 0$



the point  $(1.2, 1.2)$  maximizes the value of  $P$  given these constraints.

Which polynomials can be factored? Consider...

a) Can  $3x^2 + 7x - 12$  be factored?

b) List all integer values of  $b$  for which the trinomial  $3x^2 + bx - 12$  can be factored.

$b \in \{35, 16, 9, 5, 0\}$

	$ac = -36$
35	$(36)(-1)$
16	$(18)(-2)$
9	$(12)(-3)$
5	$(9)(-4)$
0	$(6)(-6)$
-5	$(4)(-9)$
	$\vdots$

The Common Factor Quotient (CFQ) Method of Factoring:

To factor  $ax^2 + bx + c$  using the CFQ method:

- The first step is to set up the following:  $\frac{(ax \quad)(ax \quad)}{a}$
- Find two factors of  $ac$  that have a sum of  $b$ . That is find  $e$  and  $f$  so that  $ef = ac$ , and  $e + f = b$ .
- Place  $e$  and  $f$  as shown:  $\frac{(ax+e)(ax+f)}{a}$
- Check to see if there is anything that can be factored out of the parentheses in each of the 2 sets.
- Reduce the fraction, if possible.

CFQ Example:

a)  $6x^2 + 11x + 4$        $ac = 24 (= 6 \times 4)$

$= \frac{(6x+8)(6x+3)}{6}$        $\begin{matrix} (8)(3) = 24, \\ 8+3 = 11 \end{matrix}$

$= \frac{2(3x+4)3(2x+1)}{6}$

$= (3x+4)(2x+1)$

More practice: Use CFQ to factor the following trinomials.

c)  $6x^2 + x - 2$   $ac = -12$   
 Target sum: 1  
 $(4)(-3) = -12$   
 $4 + -3 = 1$

$$\frac{(6x+4)(6x-3)}{6}$$

$$\frac{2(3x+2) \cdot 3(2x-1)}{6}$$

$$= (3x+2)(2x-1)$$

d)  $4x^2 - 11x + 6$   $ac = (4)(6) = 24$   
 target sum = -11  
 $(-8)(-3) = 24$   
 $-8 + -3 = -11$

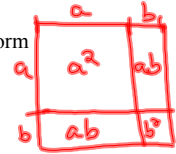
$$\frac{(4x-8)(4x-3)}{4}$$

$$= \frac{4(x-2)(4x-3)}{4}$$

$$= (x-2)(4x-3)$$

### Completing the Square

Recall: A perfect square trinomial has the form  $a^2 + 2ab + b^2 = (a+b)^2$



In the case where  $a = x$ , we have:  
 $x^2 + 2bx + b^2 = (x+b)^2$

Example: Fill in the blank in the polynomial below so that it is a perfect square trinomial. Then factor it completely.

$$x^2 + 2x + \underline{\quad}$$

$$x^2 + 2(1)x + \underline{6^2}$$

$$= x^2 + 2x + 36 = (x+6)^2$$

### Completing the Square

By "completing the square" in this way, we can solve certain quadratic equations.

Complete the square:

1.  $x^2 + 8x + 16 = (x+4)(x+4) = (x+4)^2$

note:  $\frac{1}{2}$  of 8

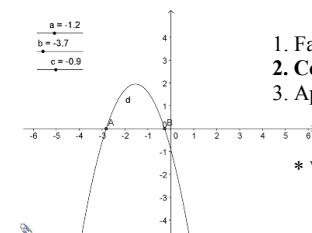
b/c  $8 = 2b \Rightarrow b = 4$

2.  $x^2 + 5x + \frac{25}{4} = (x + \frac{5}{2})(x + \frac{5}{2})$

$$\left. \begin{aligned} b &= \frac{5}{2} = 2.5 \\ b^2 &= \frac{25}{4} \end{aligned} \right\} \begin{aligned} &= x^2 + \frac{5}{2}x + \frac{5}{2}x + \frac{25}{4} \\ &= x^2 + 5x + \frac{25}{4} \end{aligned}$$

### Solving Quadratic Equations

Given  $a \neq 0$ ,  $b$ , and  $c$ , how can we find all  $x$  for which  $ax^2 + bx + c = 0$ ?



Three methods:

1. Factor and use ZPP.
2. Complete the square\*.
3. Apply the quadratic formula.

\* We'll look at this method today.

### Solving Quadratic Equations: Square Root Method

Warm-up:

- (a) Calculate  $\sqrt{25} = 5$   
(this is called the principal square root, and must be  $\geq 0$ )

- (b) Solve  $x^2 = 25$ .  $\Rightarrow x = \pm\sqrt{25} = \pm 5$   
(one solution is the principal square root of 25... the other solution is...)

### Solving Quadratic Equations: Square Root Method

The SRP (square root principle) states:

If  $x^2 = k$  and  $k \geq 0$ , then the two solutions are  $x = \pm\sqrt{k}$ .

Examples:

Solve:  $x^2 - 25 = 0$   $\Rightarrow x^2 = 25$   
 $\Rightarrow x = \pm 5$

Solve:  $(x + 3)^2 - 25 = 0$   $\Rightarrow (x + 3)^2 = 25$   
 $\Rightarrow (x + 3) = \pm 5$   
 $\Rightarrow x = -3 \pm 5$   
 $x = -3 + 5$  or  $x = -3 - 5$   
 $x = 2$  or  $-8$

### Solving Quadratic Equations: Complete the Square

Solve by completing the square and using the square root principle:

A)  $x^2 - 8x = 6$

$\Rightarrow x^2 - 8x + 16 = 6 + 16$

$b = \frac{1}{2}(-8) = -4$   
 $b^2 = 16$

$\Rightarrow (x - 4)^2 = 22$

$\Rightarrow x - 4 = \pm\sqrt{22}$

$\Rightarrow x = 4 \pm\sqrt{22}$

### Solving Quadratic Equations: Complete the Square

Solve by completing the square and using the square root principle:

B)  $2x^2 - x - 1 = 0$