

Exam #1 – Practice Problems

Counting Problems:

- Basic facts:
 - State the fundamental counting principle in your own words.
 - State the law of large numbers in your own words.
 - State the complements principle in your own words.
 - State the general (and special) additive principle in your own words.
 - How many total subsets (of any size) can be selected from an n -item set?
- Consider the word *numbers*.
 - How many different arrangements are there of the letters of the word *numbers*?
 - How many arrangements begin with a vowel?
 - How many arrangements have the word *numb* embedded in them?
- A five-card hand is dealt from a standard deck consisting of 52 cards (13 kinds and 4 suits).
 - How many different hands are possible?
 - In how many possible ways can a full house (3 of one kind, two of another kind) be dealt?
 - How about five cards of the same suit if exactly one of them is an Ace?
- Consider a 7-question true and false test. Answer these questions *without* listing all the possibilities.
 - How many different ways can a student answer the exam.
 - What is the *probability* that the student will get at least 5 correct?
- Twelve siblings (5 boys and 7 girls) must complete the Saturday chore list. There are five chores: rake the lawn, wash the dishes, clean the windows, vacuum the rug, and scrub the kitchen floor.
 - In how many ways can five siblings be chosen to do these tasks?
 - In how many ways can all girls be chosen for these chores?
- There are 5 statisticians and 6 economists to choose from.
 - In how many ways can you select a committee consisting of 2 statisticians and 2 economists?
 - What is the probability that at least one statistician is chosen for the committee (assuming they are chosen randomly)?

Probability and Expected Value Problems (with some counting mixed in):

- Basic facts:
 - State the general multiplicative principle in your own words.
 - Define the words “independent events” and “mutually exclusive events” in your own words.
 - How are odds different from probability? How do you convert from one to the other?
 - What is a ‘probability distribution,’ or ‘the distribution of a random variable’?
 - Explain how to use the probability distribution to calculate the expected value of a random variable.
- Timothy believes that if two fair coins are tossed then the probability of getting two heads should be $1/3$ because there are only three possibilities: two heads, one head, or no heads. What is the correct probability? How might the law of large numbers come into play in helping Tim understand?
- In a carnival game, you roll a pair of dice and add the values together. If the sum of the dice is anything **except** {5, 6, 7, 8, or 9}, you win \$1. If one of those sums comes up, you must pay \$1. Is this game fair, or does it favor either you or the carnival? Support your answer with appropriate calculations.

10. A red die and a blue die are rolled.
- What is the probability that the sum of the two dice is at least 8?
 - What is the probability that at least one of the dice is showing a 5?
 - What is the probability that the sum of the two dice is at least 8 given that one die is "5"?
 - Are the events "at least one die is showing a 5" and "the sum of the dice is at least 8" mutually exclusive? Independent? Both? Neither? How do you decide?
11. Three cards are drawn without replacement from an ordinary deck of 52 playing cards.
- What is the probability that the third card is a spade given the first two cards were spades?
 - What is the probability that three spades will be selected in a row?
12. There are two spinners: One is equally likely to land on 0, 1, 2, or 3 and the other is equally likely to land on 2, 3, 4, or 5.
- Use a systematic list to show the set of all possible outcomes of the two spinners. What do we call this set?

Let x equal the *sum* of the two spinners. Define the following events:

Let A be "x is odd."

Let B be " $x = 4$."

Let C be "the spinners match."

Let D be " $x > 5$."

- Which pairs of events are mutually exclusive?
 - Find the following probabilities.
 - $P(A \text{ or } B)$
 - $P(A \text{ or } D)$
 - $P(A \text{ given } D)$
 - $P(A \text{ and } D)$
 - $P(C \text{ given } D)$
 - $P(C \text{ and } D)$
 - Are A and B independent? Are C and D independent? Explain how you know.
13. A coin is tossed six times in succession.
- What is the probability distribution of the number of heads in the six tosses?
 - What is the probability that:
 - at least one head occurs?
 - exactly three heads occur?
 - more than four heads occur?
14. There are 4 gray, 6 pink, and 2 violet marbles in a hat. You pick 2 marbles from the hat. Marbles are not returned after they have been drawn. Find the probability that exactly one of the marbles is pink.
15. A game is played in which a ball is selected from one of two boxes. Box 1 contains 9 red balls and 1 white ball. Box 2 contains 4 red balls and 1 white ball. To play the game, players spin spinner to select a box: Box 1 has an 80% chance of being selected and Box 2 has only a 20% chance. Then the player reaches into the box and draws one ball at random.
- What is the probability of selecting a white ball in this game?
 - What is the probability of selecting a red ball in this game?
 - If a white ball is worth 10 points and a red ball is worth 2 points, what is the expected value for this game?