Math 151 Review - Chapter 4 Name:_____

1. **Open Sentences, Generalizations, and Existence Statements:** Classify each of the following, and write any quantifiers explicitly.

   (a) "$2x + 4 = 2(x + 2)$."

   (b) "$ax^3 + bx^2 + cx + d = 0$ has a solution."

   (c) "$x > 5 \implies x^2 > 25$."

   (d) "$(ax = ay \text{ and } a \neq 0) \implies x = y$."

   (e) "$2x + 3 = 3x + 2$ has a solution."

2. **Compound Sentences.** Identify the three sentences inherent in the following statement. How do the roles of the variables change as your focus changes from examining the two "sub-sentences" to examining the compound sentence as a whole?

$$x + a = b \iff x = b - a$$

3. **Directions:** How do the following sentences differ in terms of the intended interpretation? Which quantifiers are implied in each case?

   (a) "Solve: $x^2 + 2x + 1 = 0$."

   (b) "True or False: $x^2 + 2x + 1 = 0$."

4. **Disproving Generalizations:** Explain how you disprove a generalization. Hint: What type of sentence is the negation of a generalization?

5. **Proving Existence Statements:** Explain how you prove an existential statement. Compare your response with the previous question.

6. **Disproving Generalizations:** The following are to be interpreted as generalizations. For each of them, (i) state the negation and (ii) find a counterexample that disproves the generalization. Be sure to demonstrate that your counterexample makes the negation true! I'll be looking for this level of justification on the exam.

   (a) $x^2 > 25 \implies x > 5$.

   (b) $x < 5 \implies x^2 < 25$.

(c) $a < b \implies ax < bx$.

(d) $(x+y)^2 = x^2 + y^2$.

7. **Using Definitions:** State the negation in positive form. You may need to expand the statement using definitions. **Note: I will provide the following definitions on the exam!** You should become familiar with their meaning, however.

   - b is an upper bound for S iff $x \in S \implies x \leq b$.
   - S is bounded above iff there exists b such that b is an upper bound for S.
   - f is an increasing function iff $x < y \implies f(x) < f(y)$.

   (a) $f(x) = 1 - x$ is an increasing function.

   (b) The set $\{1, 2, 3, ...\}$ is bounded above.

   (c) $S \subset T$.

8. **Applying Theorems:** (See p. 261.) The quadratic formula reads $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In fact, the quadratic formula is the solution-pattern to a certain type of problem.

   (a) Write the problem-pattern for the quadratic formula.

   (b) Use the quadratic formula to solve for x: $x - 3x^2 = -4$.

   (c) Use the quadratic formula to solve for x: $y^2 + 2xy + x^2 = 0$.

   (d) Use the quadratic formula to solve for b: $2b^2 + bx + c = 0$.

9. **Other things you should know.** This review sheet is not comprehensive. Here are a few other things I expect you to be able to do:

   - Identify placeholders vs. free variables, and unknowns vs. parameters.
   - Know definitions of terms such as $S \subset T$, $S \cap T$, $S \cup T$. (See p. 262, Ex. 3, to see what I mean).
   - Use symbols such as $\exists$, $\forall$, s.t., etc., to appropriately write concise mathematical statements. *Please use as many quantifiers as are necessary to make the meaning clear! Friday's quiz was unique, in that I told you ahead of time not to write $\forall x$ when it was clearly implied. On the exam, I'd like you to show me that you know it is supposed to be there!*
   - And so on... let the homework and old exams (available at Cards'n'Copies) be your guide.