

Work with your neighbor! Teaching and learning go hand-in-hand...

1. **Natural Domain, Set Notation:** Express the natural domain of the following functions using at least three different set notations. Circle the answer you would be most likely to choose as a “final answer.”

(a) $\sqrt{x+3}$

(b) $\sqrt{1-x}$

(c) $\frac{x+1}{x-3}$

(d) $\frac{1}{x^2-1}$

2. **The Empty Set, and the Def'n of Subset.** Explain why the empty set ($\{\}$ or ϕ) is a subset of any other set. (Use the definitions).

3. **Function Composition.** Calculate $f(g(x))$. (Do not simplify your result.)

(a) $f(x) = x^2, g(x) = x - 1$

(b) $f(x) = \frac{1}{x+1}, g(x) = |x|$

(c) $f(x) = x(x+1), g(x) = 2x^2$

(d) $f(x) = 1 - (2x)^2, g(x) = \frac{1}{x}$

4. **Function Composition.** Find simpler functions g and h so that $f(x) = g(h(x))$. (Note: It is **not** necessary to use a third function k . Please use just two functions at first, and use a third function if you want to explore this topic a bit further.)

(a) $f(x) = \sqrt{x^2+2}$

(b) $f(x) = (x+1)^2 + 1$

(c) $f(x) = \frac{1}{1+2x}$

(d) $f(x) = (2x)^2 + 1$

Answers: 1(a) $\{x : x \geq -3\} = [-3, \infty) = (-\infty, -3)^c$, 1(b) $\{x : x \leq 1\} = (-\infty, 1] = (1, \infty)^c$, 1(c) $\{x : x \neq 3\} = (-\infty, 3) \cup (3, \infty) = \{3\}^c$, 1(d) $\{x : x^2 \neq 1\} = \{x : x \neq 1 \text{ and } x \neq -1\} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$. (2) The empty set has no members. Therefore, the definition “ $x \in \phi \implies x \in S$ ” is vacuously satisfied for any set S . 3(a) $(x-1)^2$, 3(b) $\frac{1}{|x|+1}$, 3(c) $(2x^2)((2x^2)+1)$, 3(d) $1 - (2(\frac{1}{x}))^2$. Answers may vary for number 4. Examples: 4(a) $h(x) = x^2 + 2, g(x) = \sqrt{x}$; 4(b) $h(x) = x + 1, g(x) = x^2 + 1$; 4(c) $h(x) = 1 + 2x, g(x) = 1/x$; 4(d) $h(x) = 2x, g(x) = x^2 + 1$.

5. **Solving Equations.** Justify each step in solving the following equations. Please indicate any steps where extraneous solutions may be added, and point out steps where solutions are commonly dropped. In which problems do you need to check your answer(s) at the end?

(a) $\frac{x^2 - 5x + 6}{x - 2} = 1$

(b) $\frac{x^2}{x - 3} = \frac{15 - 2x}{x - 3}$

(c) $x = \sqrt{6x - 5}$

(d) $(x + 1)(x + 4) = (2x + 1)(x + 1)$

Answers: (Note - You should show / justify each step in the solution. Here, I only discuss the key points.) (a) An extraneous solution ($x = 2$) will be found. Solution: $x = 4$. (b) An extraneous solution ($x = 3$) will be found. Solution: $x = -5$. (c) It is possible for an extraneous solution to be introduced by squaring both sides; however, this does not occur. Solution: $x = 5$ or $x = 1$. (d) Solution ($x = -1$) might be dropped when canceling the $(x + 1)$ from both sides. Solution: $x = -1$ or $x = 3$.