Math 151 Worksheet - Section 3.4 Original Copy Name: KEY

Work (and argue) with your neighbor! Teaching and learning go hand-in-hand...

First, a little new stuff...

1. What can you conclude from the compound statement: "If I go the store, then I buy ice cream, and if I buy ice cream, then I eat ice cream for dinner that night"?

   "If I go to the store, then I eat ice cream for dinner that night."

2. The *form* of the previous statement is "$A \implies B$ and $B \implies C$." (A shorthand, but slightly confusing, notation for this is "$A \implies B \implies C$.") Show, using a truth table of course, that the following is a tautology:

$$\text{“}(A \implies B \text{ and } B \implies C) \implies (A \implies C)\text{”}$$

For #3, below

A B C	A⇒B	B⇒C	① (A⇒B) and (B⇒C)	② A⇒C	①⇒②	②⇒①
T T T	T	T	T	T	T	T
T T F	T	F	F	F	T	T
T F T	F	T	F	T	T	F
T F F	F	T	F	F	T	
F T T	T	T	T	T	T	
F T F	T	F	F	T	T	Stop
F F T	T	T	T	T	T	
F F F	T	T	T	T	T	

yes, it's a tautology

3. Extend the truth table you constructed above to determine whether the converse (of "$(A \implies B$ and $B \implies C) \implies (A \implies C)$") also a tautology. Explain why or why not.

   The converse is not a tautology as the "F" in row 3 shows. (Notice there is also an "F" in row 6, but all we need is one "F" to show it is not a tautology.)

4. The term *transitive* is used to refer to operations like equality, for which $x = y$ and $y = z$ implies that $x = z$. Non-mathematicians (and mathematicians, too) use transitivity all the time, and often don't even realize it. Give several examples (besides "equals") of operations or connectives that are transitive. (Give it some thought, and then check page 194.)

   1. $(A \Rightarrow B$ and $B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ is a tautology, so "$\Rightarrow$" is transitive.
   2. $(A \Leftrightarrow B$ and $B \Leftrightarrow C) \Rightarrow (B \Leftrightarrow C)$ is a tautology, so "$\Leftrightarrow$" is transitive.
   3. (A is L.E. to B, and B is L.E. to C) $\Rightarrow$ (A is L.E. to C) is a tautology, so logical equivalence is transitive.
   4. $a > b$ and $b > c \Rightarrow a > c$

5. So what does a "tautology" tell us about the statement? Roughly, it means the statement is "always true," but that's not quite strong enough. For example,

$$x > 3 \Longrightarrow x^2 > 9$$

is "always true" (that is, it is true for all values of x, including 0, -4, and 19... right?), but it is *not* a tautology. It's not even *tautological*, which is the proper term to use in this case.

(a) See Example 3 on page 196, and explain what it means to be tautological.

A statement is tautological if its form is a tautology. (the form of "$x>3 \Rightarrow x^2>9$" is "$A \Rightarrow B$," and that's not a tautology.

(b) If a statement is always true, is it necessarily tautological?

No! See previous part. The statement may be true because of its meaning, but that doesn't make it's form tautological.

(c) If a statement is tautological, is it necessarily always true?

Yes; its form guarantees that it is always true, regardless of the specifics of the statement itself.

Do we have to keep writing "is logically equivalent to"?

No. See Theorem 3.3.9. Instead of writing "D is logically equivalent to E," we may write "$D \iff E$ is a tautology." I'm not sure if that saves us much writing, but at least now you know how "$\iff$" and "is L.E. to" are related.

What do I need to know for the exam? The following are essential parts of logic for mathematics. On pages 208 - 210, you need to know:

(a) all of page 208,
(b) DeMorgan's Laws,
(c) the theorem on "or" in the conclusion (Theorem 3.3.4),
(d) the definitions of "contradiction," "tautology," and "tautological,"
(e) which connectives are transitive, and how to express "transitivity," as in Theorem 3.4.1,
(f) Theorems 3.4.(2, 3, 4, and 5) about certain tautologies, and
(g) the Distributive Laws (Theorem 3.4.7).

Pay special attention to "'or' in the conclusion," the converse, and contrapositive, as well as DeMorgan's laws and the Distributive Laws.

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6. Do as many of the following homework problems (pg. [illegible]-201) together as time allows. Identify the *form* of the statement to verify your intuition!

(a) B7 through B12 Ex: B7 has the form "If A then (B or C)."
(b) B1
(c) B21 through B25
(d) B26 through B28
(e) A13 through A16

B7(a) We are given "B is true." Nothing more can be deduced.
(b) We are given "A and not B". Thus, we can deduce that C is true. (See "or in the conclusion").
(c) Given "not B". Nothing can be deduced.
(d) Given "not B or not C". Nothing can be deduced.
(e) Given "not B and not C." Thus, "not A".