Math 151 Worksheet - Section 4.4 Name:_____

Work (and argue) with your neighbor! Teaching and learning go hand-in-hand...

In writing Mathematics:

- Generalizations are sometimes stated in such a way that only implicitly suggest the words "for all." It is up to you to interpret the sentence correctly.
- It is especially common to omit the "for all" quantifier in statements of the form $H \Longrightarrow C$.
- Finally, it is often the case that conditional statements (i.e. "if...then..." or "$H \Longrightarrow C$ statements) are worded in such a way that the hypothesis and conclusion are obscured.

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1. **Misleading Wording:** The following sentences might be interpreted as existence statements, even though they are intended to be generalizations. Restate them so that the correct interpretation is clear.

   (a) "A product of two consecutive integers is even."

   (b) "The square of an odd number is odd."

2. **Hypothesis and Conclusion:** Restate the following sentences using "if..., then..." Underline the hypothesis once and the conclusion twice. You may (or may not) choose to introduce appropriate variables into your sentence.

   (a) "Numbers greater than 2 have squares greater than 4."

   (b) "No one passes Math151 without taking the exams."

   (c) "Multiplying a positive number by a negative number produces a negative number."

3. **Fake Conditionals:** Recall that an "if..., then..." statement should specify the hypothesis and the conclusion. The following sentences look like conditional statements, but in fact they are worded in a very misleading way. To a novice, what would the hypothesis *appear* to be? How should the sentence be stated instead? (Note: it may not even be a conditional!)

   (a) "If you compute the square of any number, then the answer is nonnegative."
   (Hint: Compare that sentence with this one: "If you go fishing at midnight on Tuesday, then the square of any number is nonnegative.")

(b) "If you add the same number to both sides of an inequality, then the direction of the inequality does not change."

4. **Context:** Sometimes you have to rely on the context to decide whether a sentence is a generalization or not. Decide whether each of the following sentences is a generalization or an existential statement, and restate the sentence making all quantifiers explicit.

   (a) "If $a \neq 0$, then $ax + b = 0$ has a solution."

   (b) "The product of two consecutive integers is 42."

   (c) "$(x + 1)^2 = x^2 + 1$."

5. **Negations:** Give the negation of the following sentences.

   (a) "Rectangles are squares."

   (b) "All sequences are either increasing or decreasing."

6. **Contrapositive:** Give the contrapositive of the following sentences.

   (a) "The product of two negative numbers is positive."

   (b) "The square root of a negative number is not a real number."