Math 151 Review - Chapter 2 Name: KEY

1. **True or False**: Examine the statements below and state whether they are true or false. If false, give a counter-example or expain why (whichever is more appropriate).

(T) (a) If $R \subset S$ and $S \subset R$, then $R = S$. ("Everything in R is in S, and vice-versa")

(T) (b) If $x \geq 0$, then $(\sqrt{x})^2 = x$.

(F) (c) $\phi - \{\phi\}$. ← has one member; has no members

(d) $(1, 3) = \{n : 1 < n < 3\}$. (F) (n represents integers; so $\{n: 1<n<3\} = \{2\}$!)

(e) $|x| < 3 \iff x^2 < 9$. (T)

(f) If $x \in S$, then $x \in S \cup T$. (T) "If x is in S then (x is in S or x is in T)".

2. **Grammatical Errors**: Something **may be** grammatically wrong with each one of the statements below. If so, what is wrong? If nothing is wrong, just say so.

(a) $x < 2$ ~~X~~ $x > -1$ "or"

(b) $S = x < 5$ this implies "S = x", but a set is not a number.

(c) $E1$ ~~X~~ $E2$ E1 and E2 are not sets!

(d) x ~~⇔~~ $x^2 \geq 0$ x is an expression, but "⇒" is used with conditional sentences!

(e) $\{-3, 3\}$ ~~⇔~~ $x^2 = 9$ same reason as above.

(f) ϕ ~~⊄~~ $\{x : x > 10\}$ subset ("⊂") would be ok, but ∅ is not a member of the set $\{x: x>10\}$.

3. **Sets**: Express the following sets using a different notation.

answers may vary:

(a) $\{2,5\}^c = (-\infty, 2) \cup (2,5) \cup (5, \infty)$

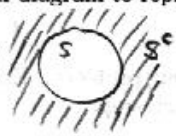
(b) $\{x : 9 - x^2 < 0\} = \{x: x^2 > 9\} = \{x: x < -3 \text{ or } x > 3\} = (-\infty, -3) \cup (3, \infty)$

(c) $(-1, 2) \cup (2, \infty)$ $\{x: -1 < x < 2 \text{ or } x > 2\}$

(d) $\{x : x > 1 \text{ and } x < 3\} = (1, \infty) \cap (-\infty, 3) = (1, 3)$.

4. **Vebb Diagrams**: Sketch a Venn diagram to represent (a) $S \cup T$, (b) S^c, (c) $S \cap T$, (d) $S^c \cap T$.

(a) 

(b) 

(c) 

(d) 

5. **Word Problems**: For each of the problems below, clearly identify all variables you choose to use. Also, when you provide an answer, you must use algebra to find it. (In other words, guessing the right answer is not sufficient.)

(a) Let x represent the length of one of the sides of a square picture frame.

i. Find a formula for the length of the diagonal.

d is diagonal length
x is the side length.

$x^2 + x^2 = d^2$ (pythagorean theorem)

$\iff 2x^2 = d^2$

ii. Find the length of one of the sides if the diagonal is 32 inches.

$32^2 = 2x^2$

$\iff 1024 = 2x^2$ (subst.)

$\iff 512 = x^2$ (uniqueness of mult.)

$\iff x = \sqrt{512}$ or $x = -\sqrt{512}$ (inverse of squaring)

So $x = \sqrt{512}$ is the solution.

(b) Betty-Sue pays no taxes on the first $12,000 of her income. Income in excess of $12,000 is taxed at 10%. If her taxes are 4% of her income, what is her income?

T = taxes, x = income

$T = .04x$, and also $T = .10(x - 12{,}000)$.

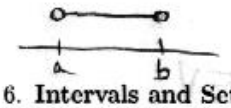
Together, $.04x = .10(x - 12000) \iff 4x = 10(x - 12{,}000)$ (u. mult.)

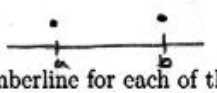
$\iff 4x = 10x - 120{,}000$ (subst.)

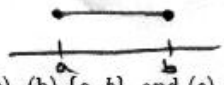
$\iff -6x = -120{,}000$ (u. add.)

$\iff x = 20{,}000$ (u. mult.)

So $x = \$20{,}000$ is her income.

(a) 

(b) 

(c) 

6. **Intervals and Sets**: Sketch one numberline for each of the sets (a) (a, b), (b) $\{a, b\}$, and (c) $[a, b]$.

7. **Sets, Subsets, and Elements**: Fill in the blank with the most appropriate of the following symbols. What other symbols (if any) would be acceptable?

$$\in \quad \notin \quad = \quad \subset$$

(a) a ___$\in$___ $\{a, b\}$

(b) a ___$\in$___ $[a, b]$

(c) a ___$\notin$___ (a, b)

(d) (a, b) ___$\subset$___ $[a, b]$

(e) $\{a, b\}$ ___$\subset$___ $[a, b]$

(f) $(a, b) \cup \{a, b\}$ ___$=$___ $[a, b]$ ($\subset$ would also be acceptable)

(g) ϕ ___$=$___ ϕ (or $\subset$)

(h) ϕ ___$\subset$___ (a, b)

8. **Functions**: Let $h(x) = 5 - 3x$. Find simpler functions f and g such that $h(x) = f(g(x))$.
$g(x) = 3x$, $f(x) = 5 - x$.

9. **Functions**: Let $f(x) = \sqrt{x+1}$, $g(x) = 4 - x$, and $A(r) = \pi r^2$.

(a) Find $A(x) + a$. $= \pi x^2 + a$

(b) Find $A(x + a)$ $= \pi(x+a)^2$

(c) Find $f(x^2)$ $= \sqrt{x^2+1}$

(d) Find $[f(x)]^2$. $= \left(\sqrt{x+1}\right)^2$

(e) Find $f(g(x))$. $= \sqrt{(4-x)+1}$

(f) Find $g(f(x))$. $= 4 - \sqrt{x+1}$

(g) State (using any correct set notation) the natural domain of each function f, g, and A.

f: $\{x : x+1 \geq 0\}$

g: $\mathbb{R}$

A: $\{r : r \geq 0\}$ (by context! radius is non-neg)

10. **Solving Equations:** Solve each of the following problems. Exhibit each step, exhibit each connective, and cite one of rules 1-9 as justification for each step.

(a) $\sqrt{x+21} = x+1$

$\Rightarrow x+21 = (x+1)^2$ (squaring)

$\Leftrightarrow x+21 = x^2+2x+1$ (subst.)

$\Leftrightarrow 0 = x^2 + x - 20$ (u. of adding twice)

$\Leftrightarrow 0 = (x+5)(x+4)$ (subst)

$\Leftrightarrow x = -5$ or $x = 4$ (zero product rule)

does not check - extraneous

(b) $\frac{1}{3}x^2 = x$

$\Leftrightarrow x^2 = 3x$ (u. mult by non-zero number)

$\Leftrightarrow x = 3$ or $x = 0$ (canceling)

- OR -

$\frac{1}{3}x^2 = x \overset{\text{(u. mult.)}}{\Leftrightarrow} x^2 = 3x \overset{\text{(u. add'n)}}{\Leftrightarrow} x^2 - 3x = 0 \overset{\text{(subst)}}{\Leftrightarrow} x(x-3) = 0$

$\Leftrightarrow x = 0$ or $x = 3$ (zero product rule)