

1. **True or False:** Examine the statements below and state whether they are true or false. If false, give a counter-example or explain why (whichever is more appropriate).

(a) If $R \subset S$ and $S \subset R$, then $R = S$.

(d) $(1, 3) = \{n : 1 < n < 3\}$.

(b) If $x \geq 0$, then $(\sqrt{x})^2 = x$.

(e) $|x| < 3 \iff x^2 < 9$.

(c) $\phi = \{\phi\}$.

(f) If $x \in S$, then $x \in S \cup T$.

2. **Grammatical Errors:** Something **may be** grammatically wrong with each one of the statements below. If so, what is wrong? If nothing is wrong, just say so.

(a) $x < 2 \cup x > -1$

(d) $x \implies x^2 \geq 0$

(b) $S = x < 5$

(e) $\{-3, 3\} \iff x^2 = 9$

(c) $E1 \subset E2$

(f) $\phi \in \{x : x > 10\}$

3. **Sets:** Express the following sets using a different notation.

(a) $\{2, 5\}^c$

(c) $(-1, 2) \cup (2, \infty)$

(b) $\{x : 9 - x^2 < 0\}$

(d) $\{x : x > 1 \text{ and } x < 3\}$

4. **Venn Diagrams:** Sketch a Venn diagram to represent (a) $S \cup T$, (b) S^c , (c) $S \cap T$, (d) $S^c \cap T$.

5. **Word Problems:** For each of the problems below, clearly identify all variables you choose to use. Also, when you provide an answer, you must use algebra to find it. (In other words, guessing the right answer is not sufficient.)

(a) Let x represent the length of one of the sides of a square picture frame.

i. Find a formula for the length of the diagonal.

ii. Find the length of one of the sides if the diagonal is 32 inches.

(b) Betty-Sue pays no taxes on the first \$12,000 of her income. Income in excess of \$12,000 is taxed at 10%. If her taxes are 4% of her income, what is her income?

6. **Intervals and Sets:** Sketch one numberline for each of the sets (a) (a, b) , (b) $\{a, b\}$, and (c) $[a, b]$.

7. **Sets, Subsets, and Elements:** Fill in the blank with the most appropriate of the following symbols. What other symbols (if any) would be acceptable?

\in \notin $=$ \subset

(a) a _____ $\{a, b\}$

(e) $\{a, b\}$ _____ $[a, b]$

(b) a _____ $[a, b]$

(f) $(a, b) \cup \{a, b\}$ _____ $[a, b]$

(c) a _____ (a, b)

(g) ϕ _____ ϕ

(d) (a, b) _____ $[a, b]$

(h) ϕ _____ (a, b)

8. **Functions:** Let $h(x) = 5 - 3x$. Find simpler functions f and g such that $h(x) = f(g(x))$.

9. **Functions:** Let $f(x) = \sqrt{x+1}$, $g(x) = 4 - x$, and $A(r) = \pi r^2$.

(a) Find $A(x) + a$.

(e) Find $f(g(x))$.

(b) Find $A(x + a)$.

(f) Find $g(f(x))$.

(c) Find $f(x^2)$.

(g) State (using any correct set notation) the natural domain of each function f , g , and A .

(d) Find $[f(x)]^2$.

10. **Solving Equations:** Solve each of the following problems. Exhibit each step, exhibit each connective, and cite one of rules 1-9 as justification for each step.

(a) $\sqrt{x+21} = x+1$

(b) $\frac{1}{3}x^2 = x$