Math 151 Review - Chapter 3 Name: KEY

1. Construct a truth table for "(A and B) or [(not A) and (not B)]." Each connective should get its own column (i.e. don't skip steps).

A	B	A and B (1)	not A	not B	not A and not B (2)	(1) or (2)
T	T	T	F	F	F	T
T	F	F	F	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	T	T

2. Give the two logically equivalent forms of $A \Longrightarrow B$. (1) not B $\Rightarrow$ not A (contrapositive)
(2) (not A) or B

3. Suppose $A \Longrightarrow B$ is a true statement. What can we say about the truth value of B?
Nothing: eg: A may be F, and F $\Rightarrow$ $\boxed{?}$ is always true. The only way we can be certain B is true is if we know A is true.

4. State the logical form of the following. Clearly indicate your choices for A, B, etc. (Remember, A, B, etc. must be true / false all by themselves. So, for instance, A cannot be "breadsticks", but it might be "I ordered breadsticks.")

(a) If $\underbrace{\text{I go to a resaurant}}_{A}$ $\underbrace{\text{I will order cheesey breadsticks}}_{B}$ and $\underbrace{\boxed{\text{I will order}}\text{ a beer}}_{C}$.
$A \Rightarrow (B \text{ and } C)$

(b) When $\underbrace{\text{it is overcast}}_{A}$ and $\underbrace{\boxed{\text{it is}}\text{ cool}}_{B}$ $\underbrace{\text{I go fishing on the Gallatin}}_{C}$.
$(A \text{ and } B) \Rightarrow C$

(c) When $\underbrace{\text{the moon passes through the Earth's shadow}}_{A}$, $\underbrace{\text{we see a lunar eclipse}}_{B}$.
$A \Rightarrow B$

(d) It is not true that, whenever $\underbrace{\text{the Boston Red Sox go to the World Series}}_{A}$, $\underbrace{\text{they lose}}_{B}$.
$\text{not}(A \Rightarrow B)$

(e) If $\underbrace{\text{today is Saturday}}_{A}$, tomorrow is not a school day.
B is "tomorrow is a school day"
$A \Rightarrow \text{not } B$

5. Formally (i.e., using A's, B's, etc.) write the contrapositive, the converse, and the negation of $A \Longrightarrow B$.
"A and not B"
"$B \Rightarrow A$"
"not $B \Rightarrow$ not A"

6. State the negation of each of the following.

(a) A and B not $(A \text{ and } B)$ is L.E. to "(not A) or (not B)" (De morgan's)

(b) $B \Longrightarrow A$ not $(B \Rightarrow A)$ is L.E. to "B and not A" (negation of a conditional)

(c) not (B and C) not(not(B and C)) is L.E. to "B and C" (double negation)

(d) A or B not $(A \text{ or } B)$ is L.E. to "(not A) and (not B)" (Demorgan's)

(e) $A \iff B$. (Tip: Rewrite this as "$(A \Longrightarrow B)$ and $(B \Longrightarrow A)$" and use DeMorgan's Law... this is a good exercise, don't just look it up!)

"not$(A \Leftrightarrow B)$" is L.E. to "not$(A \Rightarrow B$ and $B \Rightarrow A)$," which is L.E to "not$(A \Rightarrow B)$ or not$(B \Rightarrow A)$", which is L.E. to "(A and not B) or (B and not A)"

7. Compare the theorem "on two conclusions" with the theorem on "or in the conclusion".

(a) How can you tell them apart? "Two conclusions" has "and" in the conclusion; both must be true whenever the hypothesis is. But with "or in the conclusion" we only know that at least one of the two conclusions is true whenever the hypothesis is true.

(b) Give an English ~~example of both theorems.~~ Be sure your examples make sense!

"If I go fishing, then I eat chips and drink pop on the way there." That is equivalent to (by "two conclusions") If I go fishing then I eat chips and if I go fishing I drink pop."

I've only shown "two conclusions" used here. (not enough room. Sorry).

8. What is the difference between a tautology and a contradiction? How do you check to see whether a statement is either one of them? A tautology has all T's in its truth table. A conclusion has all F's. Just construct a truth table to tell if a statement is of either form.

9. Give the negation of $9 < x < 15$. (Hint: that's shorthand for a compound statement with "and" as a connective.)

"not$(9<x$ and $x<15)$" is LE to "$9 \geq x$ or $x \geq 15$."

10. Suppose "When $\overbrace{\text{a student takes a test in math class}}^{A}$, $\overbrace{\text{s/he either brings a calculator}}^{B}$ or $\overbrace{\text{s/he is good at arithmetic}}^{C}$," is true. What can be deduced from the following additional facts?

$A \Rightarrow (B \text{ or } C)$

(a) A student takes a math test but does not bring a calculator.

She is good at arithmetic.

(b) A student does not bring a calculator, and s/he is good at arithmetic.

Nothing can be deduced.

(c) A student who is good at arithmetic brings a calculator to her math test.

Nothing can be deduced.

11. Give the logically equivalent *form* (not the *name* of the form) that we studied:

(a) $A \Longrightarrow (B \text{ and } C)$ is L.E. to "$A \Rightarrow B$ and $A \Rightarrow C$" ("two conclusions")

(b) $(A \text{ or } B) \Longrightarrow C$ is L.E. to "$A \Rightarrow C$ and $B \Rightarrow C$" ("on cases")

(c) $\text{not}\,(A \text{ and } B)$ is L.E. to "not A or not B" ("Demorgan's")

12. State the *negation* of the following (simplify as much as possible):

(a) $H \Longrightarrow C$ "not $(H \Rightarrow C)$" is L.E. to "H and not C"

(b) "$\underbrace{\text{I went to the store}}_{A}$, $\underbrace{\text{and}}_{\text{and}}$ $\underbrace{\text{I ate pizza for dinner}}_{B}$."

"not (A and B)" is "(not A) or (not B)".

Here: "I did not go to the store or I did not eat pizza for dinner."

13. State the *contrapositive* of the following (simplify ~~as much~~ as possible):

(a) $A \Longrightarrow (B \text{ or } C)$ "$\text{not}(B \text{ or } C) \Rightarrow \text{not}(A)$"

"$[(\text{not } B) \text{ and } (\text{not } C)] \Rightarrow \text{not } A$"

(b) "If $\underbrace{\text{I voted on election day}}_{A}$, then $\underbrace{\text{I knew where my polling place was}}_{B}$."

"not B $\Rightarrow$ not A" is "If I did not know ... then I did not vote."

14. State the *converse* of the following (simplify as much as possible):

(a) "If a function is harmonic, then it satisfies the Cauchy-Riemann equations."

"If it satisfies Cauchy-Riemann then it is harmonic."

(b) "If today is not Saturday, then tomorrow is not Sunday."

"If tomorrow is not Sunday then today is not Saturday."

15. Prove using a truth table. Include a column for each connective (i.e. no shortcuts).

"(A and not B) $\Longrightarrow$ C" is logically equivalent to "A $\Longrightarrow$ (B or C)."

A	B	C	not B	A and not B ①	① ⇒ C	B or C ②	A ⇒ ②
T	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	F	T

these columns (circled) are the same so the statements are L.E. to each other.

16. **Contradictions:**

(a) Define "contradiction." A statement that is false in all rows of its truth table.

(b) Give a simple example of a contradiction.

A and (not A)

A	not A	A and not A
T	F	F
F	T	F

17. **Tautologies:**

(a) Define "tautology." A statement that is true in all rows of its truth table.

(b) Explain why "$x > 3 \Longrightarrow x^2 > 9$" is NOT a tautology.

The form is $H \Rightarrow C$. The truth table has an "F" in row 2. Thus, the sentence "$x>3 \Rightarrow x^2>9$" is not a tautology. (However it is true! That's because row 2 is never achieved in this example).

18. Give a common-English example of the form "$H \Longrightarrow C$" for which the *converse* is false but "$H \Longrightarrow C$" is true.

"If your pet is a german shepherd then your pet is a dog."

(The converse is clearly false: "If it's a dog then its a shepherd?"

19. Suppose the following sentence is true.

"Thoughtful college students vote on election day."

(a) State the logical *form* of the sentence. Also, clearly indicate what ~~each component~~ (A, B, etc.) stands for.

A is "you are thoughtful"

B is "you are a college student"

C is you vote on election day."

"A and B $\Rightarrow$ C"

(b) What can be logically deduced if the following facts are known *in addition to* the sentence given at the beginning of this problem? ("Nothing" is a valid response.)

i. You are a $\underbrace{\text{college student}}_{\text{B is True}}$, and you $\underbrace{\text{do not vote on election day}}_{\text{C is false}}$.

Then A cannot be true. So you are not thoughtful.

ii. You are $\underbrace{\text{not a thoughtful college student}}$.

"A and B" is false.

We can't conclude anything about C (ie. you might have voted, or might not have.)

iii. You $\underbrace{\text{do not vote on election day}}$.

Given "C" is false.

Then you cannot be a thoughtful college student. (assuming the sentence is true)

iv. You $\underbrace{\text{vote on election day}}_{\text{C is true}}$, but you are $\underbrace{\text{not a college student}}_{\text{B is false.}}$

Nothing about A (your thoughtfulness) can be deduced.

20. To the right of each of statement below, give the logical form of the statement. Use the following associations throughout this problem:

$$A \text{ is "}|x| \geq 3\text{"}$$
$$B \text{ is "}f(x) \geq 1\text{"}$$

(a) $|x| \geq 3 \Longrightarrow f(x) \geq 1$

$A \Rightarrow B$

(b) $f(x) < 1 \Longrightarrow |x| < 3$

not $B \Rightarrow$ not A

(c) $f(x) \geq 1 \Longrightarrow |x| \geq 3$

$B \Rightarrow A$

(d) $|x| < 3$ or $f(x) \geq 1$

(not A) or B

(e) $f(x) < 1 \Longrightarrow |x| \geq 3$

not $B \Rightarrow A$

(f) Which parts of (a) through (e), if any, are logically equivalent forms of "$A \Longrightarrow B$?"

a, b, and d are all L.E. to "$A \Rightarrow B$".

(contrapositive)

A	B	not A	(not A) or B
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

See!?

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21. Suppose you are given that "$A \Longrightarrow (B \text{ or } C)$" is true.

(a) Does this tell us that *both* B and C are true? Explain.

No: if A is False, nothing at all can be concluded about the "conclusion".

(b) Does this tell us that *at least one* of B or C is true? Explain.

No: see (a).