Math 151 Review - Chapter 3 Name:____________________________

1. Construct a truth table for "(A and B) or [(not A) and (not B)]." Each connective should get its own column (i.e. don't skip steps).

2. Give the two logically equivalent forms of $A \Longrightarrow B$.

3. Suppose $A \Longrightarrow B$ is a true statement. What can we say about the truth value of B?

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4. State the logical form of the following. Clearly indicate your choices for A, B, etc. (Remember, A, B, etc. must be true / false all by themselves. So, for instance, A cannot be "breadsticks", but it might be "I ordered breadsticks.")
   (a) If I go to a resaurant I will order cheesey breadsticks and a beer.
   (b) When it is overcast and cool I go fishing on the Gallatin.
   (c) When the moon passes through the Earth's shadow, we see a lunar eclipse.
   (d) It is not true that, whenever the Boston Red Sox go to the World Series, they lose.
   (e) If today is Saturday, tomorrow is not a school day.

5. Formally (i.e., using A's, B's, etc.) write the contrapositive, the converse, and the negation of $A \Longrightarrow B$.

6. State the negation of each of the following.

   (a) A and B

   (b) $B \implies A$

   (c) not $(B$ and $C)$

   (d) A or B

   (e) $A \iff B$. (Tip: Rewrite this as "$(A \implies B)$ and $(B \implies A)$" and use DeMorgan's Law... this is a good exercise, don't just look it up!)

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7. Compare the theorem "on two conclusions" with the theorem on "or in the conclusion".

   (a) How can you tell them apart?

   (b) Give an English example of both theorems. Be sure your examples make sense!

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8. What is the difference between a tautology and a contradiction? How do you check to see whether a statement is either one of them?

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9. Give the negation of $9 < x < 15$. (Hint: that's shorthand for a compound statement with "and" as a connective.)

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10. Suppose "When a student takes a test in math class, s/he either brings a calculator or s/he is good at arithmetic," is true. What can be deduced from the following additional facts?

    (a) A student takes a math test but does not bring a calculator.

    (b) A student does not bring a calculator, and s/he is good at arithmetic.

    (c) A student who is good at arithmetic brings a calculator to her math test.

11. Give the logically equivalent *form* (not the *name* of the form) that we studied:

   (a) $A \Longrightarrow (B \text{ and } C)$

   (b) $(A \text{ or } B) \Longrightarrow C$

   (c) $\text{not}\,(A \text{ and } B)$

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12. State the *negation* of the following (simplify as much as possible):

   (a) $H \Longrightarrow C$

   (b) "I went to the store, and I ate pizza for dinner."

13. State the *contrapositive* of the following (simplify as much as possible):

   (a) $A \Longrightarrow (B \text{ or } C)$

   (b) "If I voted on election day, then I knew where my polling place was."

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14. State the *converse* of the following (simplify as much as possible):

   (a) "If a function is harmonic, then it satisfies the Cauchy-Riemann equations."

   (b) "If today is not Saturday, then tomorrow is not Sunday."

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15. Prove using a truth table. Include a column for each connective (i.e. no shortcuts).

   "$(A \text{ and not } B) \Longrightarrow C$" is logically equivalent to "$A \Longrightarrow (B \text{ or } C)$."

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16. **Contradictions:**

   (a) Define "contradiction."

   (b) Give a simple example of a contradiction.

17. **Tautologies:**

    (a) Define "tautology."

    (b) Explain why "$x > 3 \Longrightarrow x^2 > 9$" is NOT a tautology.

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18. Give a common-English example of the form "$H \Longrightarrow C$" for which the *converse* is false but "$H \Longrightarrow C$" is true.

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19. Suppose the following sentence is true.

    "Thoughtful college students vote on election day."

    (a) State the logical *form* of the sentence. Also, clearly indicate what each component (A, B, etc.) stands for.

    (b) What can be logically deduced if the following facts are known *in addition to* the sentence given at the beginning of this problem? ("Nothing" is a valid response.)

        i. You are a college student, and you do not vote on election day.

        ii. You are not a thoughtful college student.

        iii. You do not vote on election day.

        iv. You vote on election day, but you are not a college student.

20. To the right of each of statement below, give the logical form of the statement. Use the following associations throughout this problem:

$$A \text{ is "}|x| \geq 3\text{"}$$

$$B \text{ is "}f(x) \geq 1\text{"}$$

(a) $|x| \geq 3 \implies f(x) \geq 1$

(b) $f(x) < 1 \implies |x| < 3$

(c) $f(x) \geq 1 \implies |x| \geq 3$

(d) $|x| < 3$ or $f(x) \geq 1$

(e) $f(x) < 1 \implies |x| \geq 3$

(f) Which parts of (a) through (e), if any, are logically equivalent forms of "$A \implies B$?"

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21. Suppose you are given that "$A \implies (B \text{ or } C)$" is true.

(a) Does this tell us that *both* B and C are true? Explain.

(b) Does this tell us that *at least one* of B or C is true? Explain.

The Most Important Logical Equivalences from Chapter 3

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22. See pages 208 thru 210 for a summary. Below, I am listing the most important *logical equivalences*. There is still much more on those three pages that you should know!

    (a) Theorem 3.2.2 “about the contrapositive”
    (b) Theorem 3.2.4 “about the converse”
    (c) Theorem 3.2.5 “on if and only if”
    (d) Theorem 3.2.6 “on two conclusions”
    (e) Theorem 3.2.7 “on hypothesis in the conclusion”
    (f) Theorem 3.2.8 “on cases”
    (g) Theorem 3.3.1 “on negation of a conditional sentence”
    (h) Theorem 3.3.2 “DeMorgan’s laws”
    (i) Theorem 3.3.5 “on ’or’ in the conclusion”

    For the theorems listed above, you need to know that the original form can be re-expressed as the logically equivalent form. For example, know that “$A \Longrightarrow (B \text{ and } C)$” can be re-written as “$(A \Longrightarrow B) \text{ and } (A \Longrightarrow C)$”.