Math 151 Review - Chapter 4 Name:________________________________

1. **Open Sentences, Generalizations, and Existence Statements:** Classify each of the following, and write any quantifiers explicitly.
   (a) "$2x + 4 = 2(x + 2)$."
   (b) "$ax^3 + bx^2 + cx + d = 0$ has a solution."
   (c) "$x > 5 \Longrightarrow x^2 > 25$."
   (d) "($ax = ay$ and $a \neq 0$) $\Longrightarrow x = y$."
   (e) "$2x + 3 = 3x + 2$ has a solution."

2. **Compound Sentences.** Identify the three sentences inherent in the following statement. How do the roles of the variables change as your focus changes from examining the two "sub-sentences" to examining the compound sentence as a whole?

$$x + a = b \iff x = b - a$$

3. **Directions:** How do the following sentences differ in terms of the intended interpretation? Which quantifiers are implied in each case?
   (a) "Solve: $x^2 + 2x + 1 = 0$."
   (b) "True or False: $x^2 + 2x + 1 = 0$."

4. **Disproving Generalizations:** Explain how you disprove a generalization. Hint: What type of sentence is the negation of a generalization?

5. **Proving Existence Statements:** Explain how you prove an existential statement. Compare your response with the previous question.

6. **Disproving Generalizations:** The following are to be interpreted as generalizations. For each of them, (i) state the negation and (ii) find a counterexample that disproves the generalization. Be sure to demonstrate that your counterexample makes the negation true! I'll be looking for this level of justification on the exam.
   (a) $x^2 > 25 \Longrightarrow x > 5$.
   (b) $x < 5 \Longrightarrow x^2 < 25$.

(c) $a < b \Longrightarrow ax < bx$.

(d) $(x+y)^2 = x^2 + y^2$.

7. **Using Definitions:** State the negation in positive form. You may need to expand the statement using definitions. **Note: I will provide the following definitions on the exam!** You should become familiar with their meaning, however.

- b is an upper bound for S iff $x \in S \Longrightarrow x \leq b$.
- S is bounded above iff there exists b such that b is an upper bound for S.
- f is an increasing function iff $x < y \Longrightarrow f(x) < f(y)$.

(a) $f(x) = 1 - x$ is an increasing function.

(b) The set $\{1, 2, 3, ...\}$ is bounded above.

(c) $S \subset T$.

8. **Applying Theorems:** (See p. 261.) The quadratic formula reads $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In fact, the quadratic formula is the solution-pattern to a certain type of problem.

(a) Write the problem-pattern for the quadratic formula.

(b) Use the quadratic formula to solve for x: $x - 3x^2 = -4$.

(c) Use the quadratic formula to solve for x: $y^2 + 2xy + x^2 = 0$.

(d) Use the quadratic formula to solve for b: $2b^2 + bx + c = 0$.

9. **Other things you should know.** This review sheet is not comprehensive. Here are a few other things I expect you to be able to do:

- Identify placeholders vs. free variables, and unknowns vs. parameters.
- Know definitions of terms such as $S \subset T$, $S \cap T$, $S \cup T$. (See p. 262, Ex. 3, to see what I mean).
- Use symbols such as $\exists$, $\forall$, s.t., etc., to appropriately write concise mathematical statements. *Please use as many quantifiers as are necessary to make the meaning clear! On the exam, I'd like you to show me that you know when "for all" is supposed to be there, even if it could be understood implicitly!*
- And so on... let the homework and old exams (available at Cards'n'Copies) be your guide.