

Ten problems, ten points each. “Technique and ability alone do not get you to the top; it is the willpower that is the most important. This willpower you cannot buy with money or be given by others..it rises from your heart” – Junko Tabei (1975, after becoming first woman to climb Everest.)

Show all work. Unjustified answers will receive zero credit.

1. Let $f(x) = \sqrt{x}$.

(a) Find the average value of f on the interval $0 \leq x \leq 4$.

(b) Find the value c in $[0, 4]$ for which $f(c)$ equals the average value of f .

2. Set up (but **DO NOT EVALUATE**) an integral for computing the volume of the solid of revolution obtained when the area between the curves $x = y$ and $x = y^2$ is revolved about the line $y = -1$. Make a sketch, and state whether you will use shells or washers.

3. A rocket weighing 3 tons is filled with 40 tons of liquid fuel. Assume that the fuel is burned off at a constant rate of 2 tons per 1000 feet of vertical height. Set up (but **DO NOT EVALUATE**) an integral that represents the total amount of work done (in foot·tons) in lifting the rocket to 3000 feet.

*Note: the weight of the rocket changes during flight as fuel is consumed. You **may** assume that the acceleration due to gravity is constant. Also, I recommend letting x represent the vertical distance the rocket has travelled, so that $x = 0$ represents the situation just before lift-off.*

4. Evaluate the following integral.

$$\int_0^{\pi} \cos^2 x \sin x \, dx$$

5. Evaluate the following integral.

$$\int 2x \sin x \, dx$$

6. Evaluate the following integral.

$$\int_0^1 \tan^{-1} x \, dx$$

7. Use an appropriate trigonometric substitution to evaluate the following integral. Be sure to express your answer in terms of x .

$$\int \frac{dx}{\sqrt{4-x^2}}$$

8. Evaluate the following trigonometric integral.

$$\int \sec^3 x \tan^5 x \, dx$$

9. Evaluate the following integral (Tip: Think partial fractions—what is the first step?)

$$\int \frac{x}{x+1} \, dx$$

10. Determine whether the following integral converges or diverges. If it converges, evaluate it.

$$\int_0^{\infty} \frac{x}{(x^2+5)^2} \, dx$$