

**M182 Section 02 – Calculus II, Summer 2002**  
**Final Exam**

Thursday, August 8, 2002

Name: \_\_\_\_\_

**Thirteen problems, 200 points total.** 55% old material, 45% new material.

If I were asked to give what I consider the single most useful bit of advice for all humanity it would be this: Expect trouble as an inevitable part of life and when it comes, hold your head high, look it squarely in the eye and say, 'I will be bigger than you. You cannot defeat me.' – Ann Landers

**Show all work unless instructed otherwise.** Unjustified answers will receive zero credit.

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1. (15 points) Evaluate  $\int x e^{-x} dx$ .

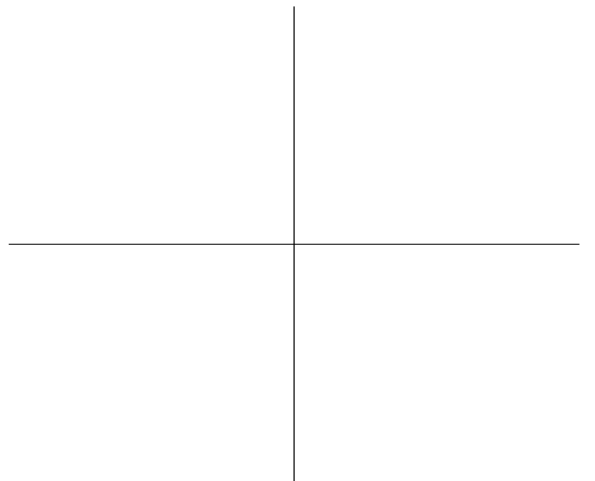
2. (15 points) Evaluate  $\int \ln x dx$ .

3. (15 points) Evaluate  $\int \tan^2 x dx$ .

4. (20 points) Evaluate  $\int \frac{\sqrt{x^2 - 25}}{x} dx$ .

5. (15 points) Set up, but **DO NOT EVALUATE**, an integral that represents the volume of the solid of revolution obtained by rotating the region between the given curves about the  $x$ -axis. You must sketch the indicated region to receive full credit.

$$y = -x^2 + 20, \quad y = \sqrt{x}, \quad x = 0, \quad x = 4$$

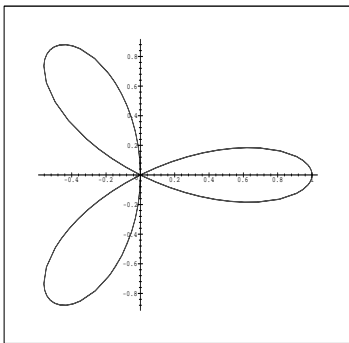


6. (15 points) Find the arc-length of the parametric curve given below:

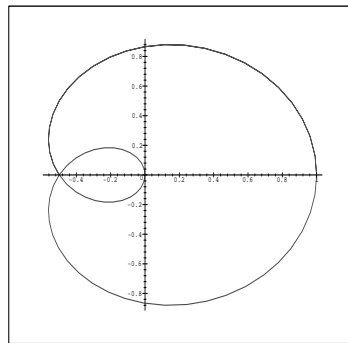
$$x = 3t - t^3, \quad y = 3t^2, \quad 0 \leq t \leq 2.$$

7. (5 points) Which of the following four choices represents the polar curve given by  $r = \cos(2\theta)$ ? You need not show your work for this problem.

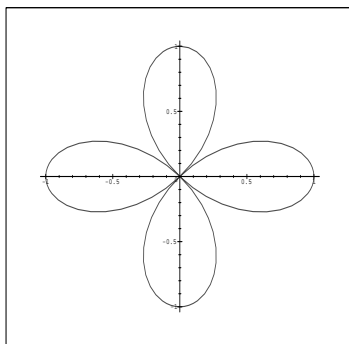
a)



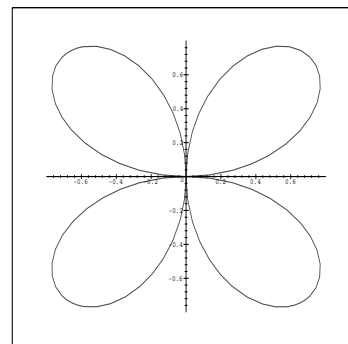
c)



b)



d)



8. (10 points) Calculate the total area enclosed by the polar curve  $r = \cos(2\theta)$ . (Note the connection with the previous question.)

9. (25 points - 5 points each) True or False (For each item below, circle either T or F. You need not show your work for this problem.)

T    F        The integral test applies to  $\sum \frac{e^k}{k}$

T    F        Since we know that  $\sum \frac{1}{k}$  is divergent, the comparison test shows that  $\sum \frac{k}{k^2 - 1}$  diverges as well.

T    F        The ratio test can be used to show that  $\sum \frac{(-1)^k}{k^2}$  converges absolutely.

T    F        If  $\sum a_k$  converges, and  $a_k > 0$  and  $b_k > 0$  for all  $k$ , and if  $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = 0$ , then  $\sum b_k$  diverges.

T    F        If  $\sum a_k$  converges, and  $a_k > 0$  and  $b_k > 0$  for all  $k$ , and  $\lim_{k \rightarrow \infty} \frac{a_k}{b_k} = 2$ , then  $\sum b_k$  diverges.

10. (15 points) Determine whether the following series is conditionally convergent, absolutely convergent, or divergent.

$$\sum_{k=0}^{\infty} (-1)^k e^{-k}$$

11. (15 points) Derive the 3rd degree Taylor polynomial approximation to the function  $f(x) = \ln x$ , centered at  $x = e$ . (Use the definition of the Taylor series.)

12. (15 points) Evaluate  $\int_0^1 \sin(x^2) dx$  using power series. Your answer may be expressed as a series. Recall that the MacLauren series for  $\sin x$  is

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \text{ for all } x.$$

13. Consider the power series  $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}(x-2)^k}{k2^k}$ .

(a) (10 points) Find the radius of convergence.

(b) (10 points) Find the interval of convergence.