M182 Lecture Worksheet, Sec. 7.2 **Name:**_____

Trigonometric Integrals. In the coming sections, we will use trig. identities to reduce complicated integrals into ones we can evaluate.

Summary of Methods:

$\int \sin^m x \ \cos^n x \, dx$	Procedure	Relevant identities
n odd:	• peel off factor of $\cos x$ • apply relevant identity • let $u = \sin x$	$\cos^2 x = 1 - \sin^2 x$
m odd:	• peel off factor of $\sin x$ • apply relevant identity • let $u = \cos x$	$\sin^2 x = 1 - \cos^2 x$
both even:	• use relevant identities to reduce power of $\sin x$ and $\cos x$	$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$ $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

$\int \tan^m x \ \sec^n x \, dx$	Procedure	Relevant identities
n even:	• peel off factor of $\sec^2 x$ • apply relevant identity • let $u = \tan x$	$\sec^2 x = \tan^2 x + 1$
m odd:	• peel off factor of $\sec x \ \tan x$ • apply relevant identity • let $u = \sec x$	$\tan^2 x = \sec^2 x - 1$
otherwise:	• use a little ingenuity • it sometimes helps to use the relevant identity to express the integral only as powers of $\sec x$	$\tan^2 x = \sec^2 x - 1$

$\int \sin(mx) \ \cos(nx) \, dx$, etc.

Procedure: These are easy! Just use the appropriate trig identity, break the integral of the sum into the sum of the integrals, and use a simple substitution (if necessary). For example:

$$\int \sin(3x) \ \cos(5x) \, dx = \frac{1}{2}\left[\int \sin(3x - 5x) \, dx + \int \sin(3x + 5x)\right] \, dx.$$

1. First let us look at integrals of the form $\int \sin^n x \ \cos^m x \, dx$. There are several cases to consider:

   (a) **Example–when the power of sine is odd:**

   $$\int \sin^5 x \ \cos^2 x \, dx$$

   (b) **Example–when the power of cosine is odd:**

   $$\int \sin^4 x \ \cos^5 x \, dx$$

(c) **Example–when both powers are even:**

$$\int \sin^4 x \ \cos^4 x \, dx$$

2. Now let us look at integrals of the form $\int \tan^m x \ \sec^n x \, dx$. There are several cases to consider:

   (a) **Example–when the power of secant is even:**

   $$\int \tan^2 x \sec^4 x \, dx$$

   (b) **Example–when the power of tangent is odd:**

   $$\int \tan^3 x \sec^3 x \, dx$$

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3. **Assigned Homework**: pg. 482, numbers 3, 9, 11, *15, *17, *19, 23, 25, *45, 51, *52, *62