M182 Lecture Worksheet, Sec. 7.3 **Name:**______________________

Trig Substitutions. The table below summarizes how trig substitutions can be used to simplify integrals that involve radicals:

Expression	Substitution	Relevant identity
$\sqrt{a^2 - x^2}$	let $x = a\sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$\cos^2 x = 1 - \sin^2 x$
$\sqrt{a^2 + x^2}$	let $x = a\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\sec^2 x = 1 + \tan^2 x$
$\sqrt{x^2 - a^2}$	let $x = a\sec\theta$, $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$	$\sec^2 x - 1 = \tan^2 x$

Note that we have made restrictions to the domain of θ so that the trig functions would be one-to-one. This allows us to express $\theta = \sin^{-1} x$, for example.

Remember that if you change variables while doing an indefinite integral, you need to **change your variable back** at the end.

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1. **Example (with $\sqrt{a^2 + x^2}$):** Evaluate $\displaystyle\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$. (Remember to change your variable back to x at the end!)

2. **Example (with $\sqrt{a^2 - x^2}$):** Evaluate the integral $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}}$.

3. **Extension:** Find the average value of $f(x) = \dfrac{1}{x^2\sqrt{4-x^2}}$ on the interval $[\sqrt{2}, 2]$.

4. **Classic Example:** Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Hint: solving for y gives

$$y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$$

5. **Extension:** Interpret your answer for the special case when $a = b$.

6. **Example (completing the square):** $\int \frac{1}{\sqrt{9x^2+6x+2}}\,dx$

7. **Homework**: pg. 488-489, numbers 1, 2, 3, 4, 5, 7, 8, 12, 13, 25, 33