M182 Lecture Worksheet, Sec. 8.2 **Name:**____________________________

Surface Area. To calculate the surface area of a surface of revolution, we approximate the curve by the differential ds as we did when computing arc length in Section 8.1. We then revolve the "approximated curve" about the desired line to obtain an approximation for the surface.

We can think of revolving the differential ds about the desired line as forming a thin ribbon (actually, a **frustrum** of a cone), whose surface area is given by $2\pi r\,ds$, where r is the radius of the ribbon. Therefore, we integrate to find the total surface area:

$$\text{Surface Area} = \int_a^b 2\pi r\,ds$$

Recall from the previous section that the Pythagorean Theorem gives $ds = \sqrt{1+(\frac{dy}{dx})^2}\,dx$ (or, if we prefer, $ds = \sqrt{1+(\frac{dx}{dy})^2}\,dy$). When we revolve the curve $y = f(x)$ about the x-axis, the radius is y and the surface area formula becomes

$$\int_a^b 2\pi y\,ds$$

If we instead revolve the curve $y = f(x)$ about the y-axis, the radius is x and so the surface area formula becomes

$$\int_a^b 2\pi x\,ds$$

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1. **Example**: Find the area of the surface generated by revolving $y = x^3$ (with $0 \leq x \leq 1$) about the x-axis.

2. **Example (Surface area of a sphere)**: Compute the surface area of a sphere by revolving the curve $y = \sqrt{r^2 - x^2}$ about the x-axis.

3. **Example**: Set up (do not evaluate) the integral for the area of the surface obtained by rotating the curve $x = e^{2y}$, $0 \leq y \leq \frac{1}{2}$, about the y-axis.

4. **Example (Gabriel's Horn)**:

   (a) Show that the **volume** of the solid of revolution obtained by rotating the region bounded by the curves $y = 1/x$, $x = 1$, and $y = 0$ about the x-axis is finite. (This object is known as Gabriel's Horn.)

   (b) Show that the **surface area** of the object is not finite. (So Gabriel's Horn cannot hold enough paint to coat its surface!)

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5. **Assigned Homework**: pg. 552-553, numbers 1, 5, 11, *27