M182 Lecture Worksheet, Sec. 10.5 **Name:**______________________________

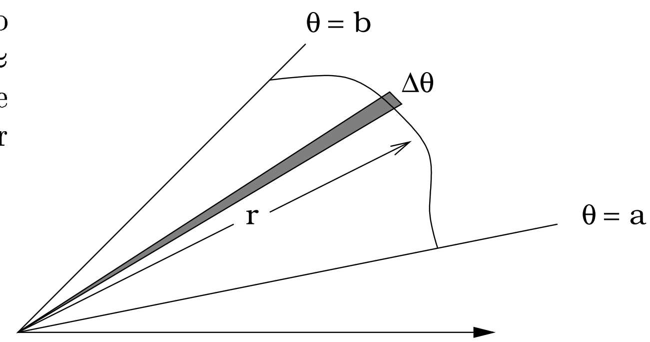
Part 1: Areas in Polar Coordinates. Recall that the area of a sector of a circle with central angle θ and radius r is given by $\frac{1}{2}r^2\theta$, because

$$\underbrace{\pi r^2}_{\substack{\text{area of}\\ \text{full}\\ \text{circle}}} \cdot \underbrace{\frac{\theta}{2\pi}}_{\substack{\text{percent of}\\ \text{full circle}}} = \underbrace{\frac{1}{2}r^2\theta}_{\substack{\text{area of}\\ \text{sector}}}$$

We can divide the area of a polar region up into many small sectors, each of which has area $\Delta A_i \approx \frac{1}{2}r^2\Delta\theta$. Summing all of these up and letting the number of sectors go to ∞ gives the formula for the area of the polar region,

$$\text{Area of Polar Region} = A = \int_a^b \frac{1}{2}r^2\, d\theta$$

where θ ranges from a to b, and $a < b < (a + 2\pi)$. (The restriction on a and b ensures that we only go around the region once).



Often the hardest part of doing these problems is determining the limits of integration. It is usually necessary to sketch the curve(s) first.

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1. **Example:** Find the area of the portion of the cardioid $r = 1 + \sin\theta$ that lies in the 3rd quadrant.

2. **Example:** Find the area of the four-leaved rose $r = \cos(2\theta)$.

3. **Example:** Sketch and find the area of the region $0 \le r \le \theta$, where $0 \le \theta \le 2\pi$.

4. **Example (Area between Two Curves):** Find the area of the region that lies inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$.

Part 2: Arc Length of Polar Curves. To compute the arc length of a polar curve where $a \le \theta \le b$, we use the parameterization

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

and use our old arc length formula:

$$L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

The example below asks you to work out the following arc length formula for polar curves, which follows from the old arc length formula given above.

$$\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

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5. **Example:** Compute $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ and apply our old arc length formula to derive the polar arc length formula given above.

6. **Example:** Find the arc length of the cardioid $r = 1 + \sin\theta$.

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7. **Assigned Homework**: pg. 674, numbers 3, 5, 7, 11, *15, *21, *27, 31, *35, 45, 47, *53