M182 Lecture Worksheet, Sec. 10.7. **Name:**____________________________

Conic Sections in Polar: We can use polar coordinates to devise a simple method for universally categorizing ellipses, hyperbolas, and parabolas under one system, including those that have been rotated! All we need is to fix the focus (or one of the foci) at the origin, and the equations we obtain are easy to work with.

Theorem 1: Let F be any fixed point (focus) and let l be any fixed line (directrix). Then the set of points

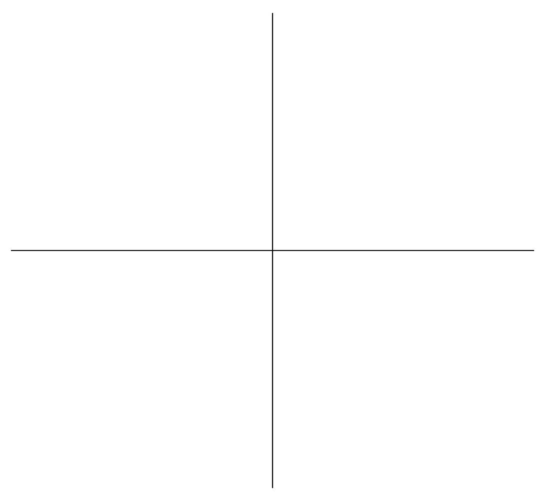
$$\frac{|PF|}{|Pl|} = e$$

is a conic section. The number e is called the **eccentricity** of the conic section. Morover, the conic is

1. an ellipse if $e < 1$
2. a parabola if $e = 1$
3. a hyperbola if $e > 1$.

Proof:

Case 1: Suppose $e = 1$. Then...



Case 2: Suppose $e \neq 1$. Then $|PF| = e|Pl|$. Note that in polar coordinates,

$|PF| =$

and $|Pl| =$

So $|PF| = e|Pl|$ can be written:

GOAL: $\dfrac{(x-h)^2}{a^2} + \dfrac{y^2}{b^2} = 1$

OR: $\dfrac{(x-h)^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Now square both sides:

Convert to rectangular coordinates:

Group like terms:

Complete the square and simplify the right-hand side:

Divide both sides by the right-hand side: (NOTE $e \neq 1$)

Now draw conclusions by matching the resulting equation up with the standard equations for an ellipse and a hyperbola in rectangular coordinates.

Consider $e < 1$:

Consider $e > 1$:

By solving the equation $|PF| = e|Pl|$ (that you obtained on the previous page) for r, we obtain the following theorem.

Theorem 2: A polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \qquad \text{or} \qquad r = \frac{ed}{1 \pm \sin\theta}$$

represents a conic section with eccentricity e. The conic is an ellipse if $e < 1$, a parabola if $e = 1$, and a hyperbola if $e > 1$.

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1. **Example:** A conic is given by the polar equation $r = \dfrac{10}{3 - 2\cos\theta}$. Find the eccentricity, identify the conic, locate the directrix, and sketch the conic.

2. **Example:** What happens to the conic section, above, if you replace θ with $(\theta - \pi/4)$?