Math 182: Convergence and Divergence of Series

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1. At a glance, does the limit of the term go to zero? If not, then use divergence test.

   Divergence Test: If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

2. Is the series exactly a p-series or a geometric series? If so, use appropriate check.

   $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$

   $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $|r| < 1$, otherwise diverges.

3. Does the series look like a p-series or a geometric series? If so, use a comparison test.

   The Comparison Test: Suppose that $\sum a_k$ and $\sum b_k$ are series with positive terms, and that eventually $a_k \leq b_k$ for all k.

   (a) If $\sum b_k$ is convergent then so is $\sum a_k$.

   (b) If $\sum a_k$ is divergent then so is $\sum b_k$.

   Limit comparison test: Suppose $\sum a_k$ and $\sum b_k$ are series with positive terms. If $0 < \lim_{k\to\infty} \frac{a_k}{b_k} < +\infty$, then either both series converge or both diverge.

4. If the series alternates, then use the alternating series test.

   Alternating Series Test: The series $\sum(-1)^k a_k$ converges

   (a) $a_k \geq a_{k+1} \geq 0$ for large k, and

   (b) $\lim_{n\to\infty} a_n = 0$

5. If the series contains factorials and/or products involving powers, then ratio test is a good candidate. If the series is a (function of k) raised to the kth power, then the root test is a good choice.

   Ratio Test: Let $\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = L$. **Root Test:** Let $\lim_{k\to\infty} \sqrt[k]{|a_k|} = L$.

   (a) If $L < 1$, then the series converges absolutely.

   (b) If $L > 1$, then the series diverges.

6. When all else fails, try integral test.

   Integral Test. Suppose $f(x)$ is continuous, positive, decreasing function on $[a, \infty)$ with $a_k = f(k)$. Then

   $$\sum_{k=1}^{\infty} a_k \text{ and } \int_a^{\infty} f(x)\,dx$$

   either both converge or both diverge.