M182 Section 02 – Calculus II, Summer 2003 Name:______________________________

Exam II Tuesday, July 29, 2003

Seven problems, 100 points total.

> "Most of the important things in the world have been accomplished by people who have kept on trying when there seemed to be no hope at all." - Dale Carnegie.

Show all work. Unjustified answers will be treated with skepticism.

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1. (12 points) Consider the parametric equations

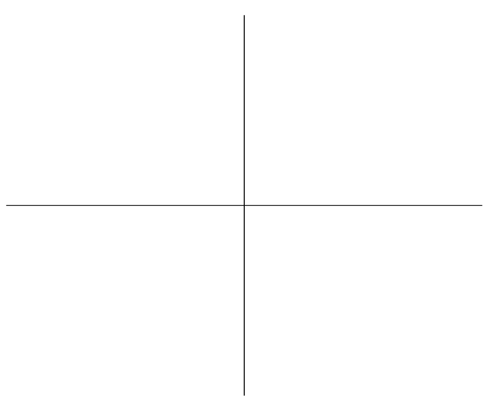
$$x = 1 + e^{2t}, \qquad y = e^t, \qquad -\infty \leq t \leq \infty$$

(a) Eliminate the parameter to obtain a cartesian equation.

(b) Find the points (x, y) corresponding to $t = -\infty$ and $t = 0$.

(c) Find the slope of the tangent line at the point corresponding to $t = 0$.

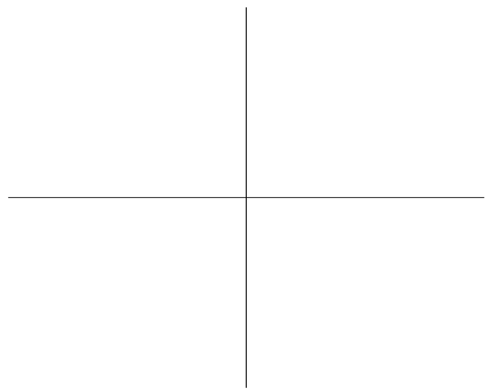
(d) Use the information above to sketch the parametric curve. Be sure to indicate the orientation of the curve and to label the points corresponding to $t = -\infty$ and $t = 0$.



2. (10 points) Suppose

(i) $c > 0$
(ii) $y = f(x)$ for all $a \leq x \leq b$, and
(iii) $f(x) \leq c$ for all $a \leq x \leq b$.

(a) Sketch the surface of revolution that would be obtained by rotating such an f about the horizontal line $y = c$.



(b) Set up an integral that will give the area of the surface of revolution.

3. (10 points) Consider the parametric equations

$$x = 1 + 3\cos(t), \qquad y = \sin(t) - 5, \qquad 0 \le t \le 2\pi$$

(a) There are two points on the parametric curve with a horizontal tangent. Find one of them. Report your answer as (x, y) coordinates.

(b) There are also two points with a vertical tangent. Find one of them. Report your answer as (x, y) coordinates.

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4. (40 points total) Indicate whether the following sequences and series are convergent or divergent.

For full credit, you must justify your answer. If you used any "convergence tests", you must list the hypotheses. However, you need only convince yourself (not me) that the hypotheses are satisfied.

(5 points each for (a) and (b))

(a) (converge) (diverge) The sequence $\{e^{-n} + 1\}_{n=0}^{\infty}$
<u>Justification:</u>

(b) (converge) (diverge) The sequence $\left\{\dfrac{n+1}{n}\right\}_{n=1}^{\infty}$
<u>Justification:</u>

(6 points each for (c) thru (g))

(c) (converge) (diverge) The series $\sum_{k=0}^{\infty}(e^{-k}+1)$

Justification:

(d) (converge) (diverge) The series $\sum_{k=1}^{\infty}\frac{2^k}{k!}$

Justification:

(e) (converge) (diverge) The series $\sum_{k=1}^{\infty}\frac{k^2+1}{(k^2-1)(k^2-2)}$

Justification:

(f) (converge) (diverge) The series $\sum_{k=2}^{\infty}\frac{1}{\ln k}$

Justification:

(g) (converge) (diverge) The series $\sum_{k=0}^{\infty}\frac{(-1)^k k}{k^2+1}$

Justification:

5. (10 points) Show that the following sequence is eventually decreasing:

$$a_n = \frac{\ln\left(\frac{n}{2}\right)}{n}$$

6. (8 points) Calculate the value of the following series, or explain why it is divergent.

$$\frac{5}{4} + \frac{15}{16} + \frac{45}{64} + \frac{135}{256} + \ldots$$

7. (10 points total) **TRUE** or **FALSE**. You do NOT need to justify your answer to this part. (2 points each).

   (a) TRUE or FALSE: A series can be both convergent and absolutely convergent.

   (b) TRUE or FALSE: The Divergence Test can be used to show that the sequence $\left\{\left(\frac{2k+1}{3k}\right)\right\}_{k=0}^{\infty}$ diverges.

   (c) TRUE or FALSE: The alternating harmonic series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}$ is conditionally convergent.

   (d) TRUE or FALSE: $\sum_{k=0}^{\infty} \frac{1}{2^k}$ is a convergent p-series.

   (e) TRUE or FALSE: If $f(n) = a_n$ for all n and $\int_1^{\infty} f(x)\,dx$ is convergent, then $\sum_{k=1}^{\infty} a_n$ is also convergent.

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8. **Extra Credit** (5 points). Suppose $(x(t),\ y(t))$ defines the motion of a particle in the x-y plane as a function of the time parameter t. Explain how to compute the speed of the particle. (Note: the answer is NOT "compute $\frac{dy}{dx}$".)