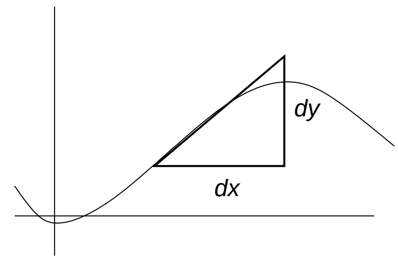
M182 Lecture Worksheet, Sec. 5.5 **Name:**______________________________

Simple Substitution: The most basic technique of integration is called the method of simple substitution. As with most of the more advanced integration techniques that you will learn in this course, the idea is to make a change of variable to simplify the integrand.

Recall that $\frac{dy}{dx} = f'(x)$ represents the slope of the tangent line to the curve $y = f(x)$. So the "fraction" $\frac{dy}{dx}$ can be interpreted as "rise over run" as in the picture. We will find this useful throughout the course as a device for understanding the concepts we discuss. Notice that if we treat dy and dx as separate **differentials**, then we see that



$$\frac{dy}{dx} = f'(x) \qquad \text{implies that} \qquad dy = f'(x)\,dx$$

(Manipulating differentials in this way will be very common in this course, as well as in Diff. Eq. and Multivariate Calculus).

The Method of Simple Substitutions. If $u = g(x)$, then $du = g'(x)dx$ (as discussed above). Therefore,

$$\int f(\underbrace{g(x)}_{u}) \cdot \underbrace{g'(x)\,dx}_{du} = \int f(u)\,du$$

Ideally, by making the substitution $u = g(x)$, we end up with an integral $\int f(u)\,du$ that is easy to integrate.

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1. **Example:** Calculate the derivative of $y = (1 + \sin x)^3$.

2. **Extension:** What is $\int 3(1 + \sin x)^2 \cos x\,dx$?

3. **Extension:** Let $u = 1 + \sin x$. Then $du =$ ____________. Rewrite the integral in terms of u.

4. **Example:** Note that $\int \cos x \sin x \neq \int \cos x\,dx \cdot \int \sin x\,dx$. (How can you tell?) Use a simple u-substitution to evaluate the integral:

   (a) Let $u = \sin x$

   (b) Let $u = \cos x$

   (c) Why did you get different answers?

5. **Example:** $\int x^3 \cos(x^4 + 2)\,dx$

6. **Example:** $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$.

   (a) Try letting $u = \sin x$.

   (b) Try letting $u = \cos x$.

7. **Examples:**

(a) $\int x(4+x^2)^{10}\, dx$

(b) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

(c) $\int e^{\sin\theta}\cos\theta\, d\theta$

(d) $\int \frac{3x-1}{(3x^2-2x+1)^4}\, dx$

(e) $\int \frac{x}{x^2+1}\, dx$

(f) $\int x(x^2+1)^{3/2}\, dx$

8. **Example (Substituting for x)**: $\int \frac{x}{1+x}\, dx$.

Dealing with Definite Integrals: When doing a simple substitution with a definite integral of the form $\int_a^b f(g(x))\cdot g'(x)\, dx$, you have two options:

Option 1. Find an antiderivative $F(x)$ by using a u-substitution on $\int f(g(x))\cdot g'(x)\ dx$ as above, and then apply the fundamental theorem of calculus to evaluate the definite integral: $F(b)-F(a)$.

Option 2. Change the limits of integration and evaluate $\int_{u(a)}^{u(b)} f(u)\, du$.

9. **Example:** $\int_0^4 \sqrt{2x+1}\, dx$. (Use both options.)

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10. **Assigned Homework:** Page 416-417, numbers 4, 5, 7, 11, 21, 31, 35, 41, 49, 55, 63, 69