M182 Lecture Worksheet, Sec. 11.8 **Name:**____________________________

Power Series. A power series can be thought of as an infinitely long polynomial. Formally, a power series is any series of the form

$$\sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$$

Here, x is a variable, and the c_k's are the coefficients of the power series. Notice that for fixed values of x, the power series is just a series of constants, so all of our old tests for convergence still work.

The power series should be thought of as a function $f(x) = \sum_{k=0}^{\infty} c_k x^k$.

Note that a power series may converge for some values of x and diverge for others. To make sure the function is well defined, we restrict the domain of the function to those values of x for which the series converges. More generally,

$$f(x) = \sum_{k=0}^{\infty} c_k (x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots$$

is a **power series centered at** a.

Whenever we mention a power series (particularly in the next sections when we talk about using them to approximate other functions), we need to be careful to mention the radius of convergence and interval of convergence of the power series.

We say that if the power series centered at a converges for $-R < |x-a| < R$ then the **radius of convergence** is R, and the **interval of convergence** is $(a-R, a+R)$. It is possible that the interval of convergence may include none, one or both of its endpoints. The Ratio Test (and sometimes the Root Test) is useful for finding the interval of convergence.

Fact: The radius of convergence of a power series is always either zero, infinity, or some finite positive constant.

---

1. **Key Example:** Let $f(x) = \sum_{k=0}^{\infty} c_k x^k$, where $c_k = 1$ for all k.

   (a) Find the domain of the function f, and find a simple expression for $f(x)$ on its domain.

   (b) What do the partial sums look like? Write out $s_2 = \sum_{k=0}^{1}$ and s_5.

   (c) Use a calculator to graph the function you found in part (a), as well as s_2 and s_5. What happens to the graph of s_n as $n \to \infty$?

2. Find the radius and interval of convergence of the series $\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$.

3. Find the interval and radius of convergence of

$$\sum_{k=1}^{\infty} \frac{(x-1)^k}{k} = (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4} + \ldots$$

4. Find the interval and radius of convergence of $\sum_{k=1}^{\infty} k!x^k$.

5. Given that $\sum_{k=0}^{\infty} c_k x^k$ converges for $x = 3$ but diverges when $x = 4$, what can you conclude about the convergence of the following series?

   (a) $\sum_{k=0}^{\infty} c_k(-2)^k$

   (b) $\sum_{k=0}^{\infty} c_k 5^k$

   (c) $\sum_{k=0}^{\infty} c_k(-3)^k$

---

6. **Homework**: pg. 744, numbers 1, 2, 3, 5, 7, 9, 17, 19, 27, 29, 30