

Last time...

There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference.

How many different ways are there to select a group of four students to attend the conference?

$$C(37, 4) = \frac{37 \cdot 36 \cdot 35 \cdot 34}{4!}$$

Sep 8-8:58 AM

There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference.

If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?

$$\frac{C(14, 2)}{j's} \times \frac{C(23, 2)}{sr's}$$
$$\frac{14 \times 13}{2!} \times \frac{23 \times 22}{2!} =$$

Question: How is the fundamental counting principle being applied?

Sep 8-8:58 AM

Sec. 11.3 Continued - Tying it all together

Permutations: $P(n,r) = n(n-1)(n-2)\dots(n-r+1)$
 ...connect to the $n! / (n-r)!$ rule. $10(9 \times 8) \leftarrow$
 $10-3+1$

$$P(10,3) = \frac{10 \times 9 \times 8 \times 7!}{7!}$$

Combinations: $C(n,r) = \frac{P(n,r)}{r!}$
 ...why divide by $r!$?

$$C(10,3) = \frac{10!}{3!(7!)} = \frac{10 \times 9 \times 8}{3!} \quad \begin{matrix} P(10,3) \\ 4 \times 3 \times 2 \times 1 \end{matrix}$$

Is $P(10, 3) = P(10, 7)$?

$$\frac{10!}{7!} \neq \frac{10!}{3!}$$

Is $C(10, 3) = C(10, 7)$?

$$\frac{10!}{3!(7!)} = \frac{10!}{7!(3!)}$$

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#23 - Sum of digits

(How many counting numbers have four distinct nonzero digits such that the sum of the digits is 10? 11?)

a) $\underline{4} + \underline{3} + \underline{2} + \underline{1} = 10$

$$P(4,4) = 4! = 24$$

b) 5, 3, 2, 1

$$P(4,4) = 24.$$

Sep 9-9:00 AM

#25 - Defective CD Players.

(If a shipment of 24 DVD players contains six defective ones, how many 5-item samples would not include any of the defective ones?)

$$\underline{18} \cdot \underline{17} \cdot \underline{16} \cdot \underline{15} \cdot \underline{14}$$

~~$P(18, 5)$~~

$$C(18, 5) = \frac{18!}{5!(13!)} = \dots$$

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#27 - Choosing poker hands

How many ways are there to select a hand with...

- (a) all diamonds? $\rightarrow C(13, 5)$
- (b) all black cards? $\rightarrow C(26, 5)$
- (c) all aces? $\rightarrow \cancel{C(4, 5)}$ not possible

$$\overline{D_1} \quad \overline{D_2} \quad \overline{D_3} \quad \overline{D_4} \quad \overline{D_5}$$


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#29 - New home models

Jeff is building six homes on a block.

(a) How many ways can he arrange them if there are 3 standard homes and 3 deluxe homes? (All standard homes are identical, as are all deluxe homes)

$$C(6,3) = \frac{6!}{3!(3!)} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$$



$$\frac{6 \times 5 \times 4}{3!} = P(6,3)$$

* need to divide by 3! to account for the redundant rearrangements that produce the same pattern on the block.

(b) What if there are only two deluxe homes and 4 standard homes?

$$C(6,2) = \frac{6 \times 5}{2} = 15$$
$$C(6,4) = \frac{6 \times 5 \times 4 \times 3}{4!} = 15$$

$$C(6,2) \times C(4,4) = 15 \times 1 = 15$$

Sep 9-9:16 AM

Combining permutations, combinations, and the fundamental counting principle.

Some problems require us to combine strategies. For instance:

←-----→
* How many different 5-card poker hands contain two pairs (that's 2 cards of one denomination, 2 cards of another, and 1 card of a third denomination)?

We can break this task into steps:

1. choose the two denominations for the two pairs $\rightarrow \frac{13 \times 12}{2}$
2. choose the 1st pair $C(4,2)$
3. choose the 2nd pair $C(4,2)$
4. choose the remaining card 44

$$\text{FCP: } C(13,2) \times C(4,2) \times C(4,2) \times 44 = 123,552$$

Sep 9-9:23 AM