Mth126 - Homework Day!

Announcements / Reminders:

- Quiz on Thursday focused on 12.1 - 12.3
- Tomorrow we'll start 12.5: Expected Value
  (skipping 12.4 on Binomial Prob)
- Homework Set #1 is due Monday

Sep 17-9:00 AM

A bucket contains black chips and white chips. A person selects two chips without replacement.

If the probability of selecting a black chip and then a white chip is 15/56, and the probability of selecting a black chip on the first draw is 3/8, find the probability of selecting the white chip on the second draw, given that the first chip selected was black.

Can you find two different ways?

$P(B_1 \cap W_2) = \frac{15}{56}$

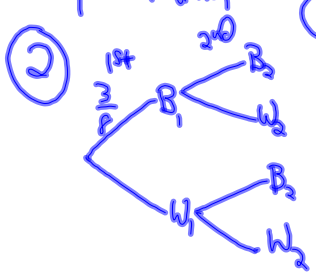
$P(B_1) = \frac{3}{8}$

Find: $P(W_2 | B_1) = ?$

① $P(B_1 \cap W_2) = P(B_1) \times P(W_2 | B_1)$

$\frac{8}{3} \cdot \frac{15}{56} = \frac{3}{8} \times P(W_2 | B_1) \cdot \frac{8}{3}$

$\frac{5}{7} = P(W_2 | B_1)$

② 

HW Finish by weighting branches w/ probabilities and use it to answer the question.

Sep 17-9:02 AM

A glass jar contains 5 red, 3 blue and 2 green balls. Two balls will be drawn without replacement from the jar.

a) What is the probability that both are blue?

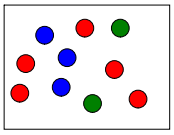
(1) $P(B_1 \cap B_2) = P(B_1) \cdot P(B_2 | B_1)$

(2) $= \frac{C(3,2)}{C(10,2)}$ $\frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{3}{45} = \frac{1}{15}$

b) What is the probability that exactly one is blue?

$P((B_1 \cap B_2') \cup (B_1' \cap B_2)) = P(B_1 \cap B_2') + P(B_1' \cap B_2)$ (by Special Addition Rule)

(Blue on 1st and not Blue on 2nd) or (not Blue on 1st and Blue on 2nd)

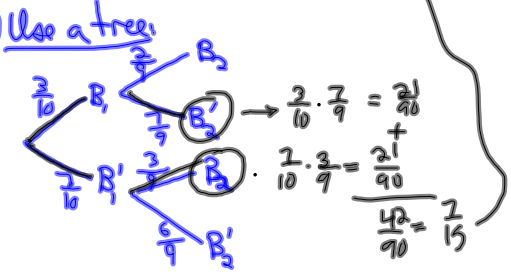


$= P(B_1) \cdot P(B_2' | B_1) + P(B_1') \cdot P(B_2 | B_1')$

$= \frac{3}{10} \cdot \frac{7}{9} + \frac{7}{10} \cdot \frac{3}{9}$

$= 42/90 = \frac{7}{15}$

(2) Use a tree:



Sep 17-9:03 AM

Sec. 12.3 #33: If one number is chosen randomly from the set {1, 2, ...10}, then

P(odd and prime) = P(odd) * P(prime | odd)
= 5/10 * 3/5
= 3/10.

Compare that with P(prime) * P(odd | prime). What do you learn?

$\frac{4}{10} \times \frac{3}{4} = \frac{3}{10}$

Notice that $P(\text{prime} \mid \text{odd}) = P(\text{odd} \mid \text{prime})$

This suggests the following:

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

$P(B \text{ and } A) = P(B) \cdot P(A|B)$

Sep 17-9:07 AM

Sec. 12.3 #28

a) P(A and B) = P(A) * P(B|A) (*by definition*)

b) P(B|A) = P(A and B) / P(A)

(solve (a) for P(B|A))

c) P(B|A) = n(A and B) / n(S)
n(A) / n(S)

$\frac{n(S)}{n(S)}$ (def'n of probability: # favorable / # in sample space)

d) P(B|A) = n(A and B)
n(A)

→ mult. by $\frac{n(S)}{n(S)}$.

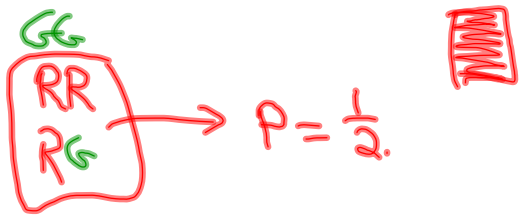
Sep 17-9:12 AM

Sec. 12.3 #24. Two cards are dealt from a 52 card deck. Find P(club 2nd | diamond 1st).

after 1 diam., 13 clubs are left, out of 51 cards total. So $P = \frac{13}{51}$.

Sep 17-9:04 AM

12.3 #62. There are three cards: green/green, red/red, and red/green. One of the cards is selected randomly and laid on the table. If it happens to have a red side up, what is the probability that it is also red on the other side?



Sep 17-9:10 AM

12.3 #67-70.

Four-engine plane.

Let E_1 be the event that engine 1 fails (etc.)

Each engine has 10% chance of failure if all 4 are running.

...20% chance of failure if only 3 are running.

...30% chance of failure if only 2 are running.

Plane can fly with as few as 2 engines running.

67. Find P(no engine failures) $= P(E_1' \cap E_2' \cap E_3' \cap E_4')$

$= .9 \times .9 \times .9 \times .9 = .6561$

68. Find P(exactly one failure)

$= 4 \times P(E_1 \cap E_2' \cap E_3' \cap E_4')$

$= 4 \times (.10 \times .80 \times .80 \times .80)$

$= 4(.0512) = .2048$

Sep 17-9:17 AM

12.3 #67-70 (cont).

Four-engine plane.

Each engine has 10% chance of failure if all 4 are running.

...20% chance of failure if only 3 are running.

...30% chance of failure if only 2 are running.

Plane can fly with as few as 2 engines running.

69. Find P(exactly two failures)

4 engines | How many ways could 2 of them fail? | $C(4,2)=6$ ways

$\rightarrow 6 \times P(E_1 \cap E_2 \cap E_3' \cap E_4')$

$= 6 \times (.10 \times .20 \times .70 \times .70)$

$= 6 \times (.0098) = .0588$

70. Find P(failed flight)

$= 1 - P(\text{successful})$

$= 1 - P(0 \text{ fail} \cup 1 \text{ fail} \cup 2 \text{ fail})$

$= 1 - (.6561 + .2048 + .0588)$

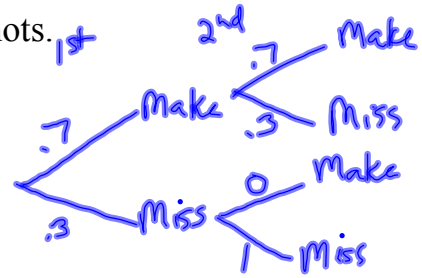
$= .0803$

Sep 17-9:26 AM

12.3 #71-73.

A one-and-one free throw situation involves one shot for sure, and a second shot is granted if the first shot goes in. Each basket counts as 1 point, for a possible scores of 0, 1, or 2 pts.

Susan makes 70% of her foul shots.



71. Find P(no points) $= \frac{3}{10}$

72. Find P(one point) $= .21$

73. Find P(two points) $= .49$

Sep 17-9:26 AM

12.2 #15. Find the probability of drawing a single card from a 52-card deck that is "neither a heart nor a ~~spade~~ seven."

not (heart) and not (seven)

~~7C~~
~~SH~~
~~7H~~

Suggestion:

1 - P(heart or seven)

Sep 17-9:28 AM

Wrap-up:

* No new homework tonight.
* Remember, Quiz tomorrow.
* Remember, Homework Set due Monday.
* Office Hours 1-2pm today, but I'll be around all afternoon.
* Homework solutions to 12.2 and 12.3 should be posted by this afternoon.

Reflect:

* How can the rule for P(A and B) = P(A) * P(B|A) be understood / explained using a tree diagram?

Have a great day!

Sep 17-9:34 AM