

## 12.5 - Expected Value

HW # 1, 3, 5, 7-9, 11, 12

Warm-up: If a carnival game has a 30% chance of paying nothing, a 50% chance of paying \$1, and a 20% chance of paying \$5, how much should the carnival charge if they wish to just break even?

if 100 play...

$$\begin{array}{r} 30 \text{ win } \$0 = \$0 \\ 50 \text{ win } \$1 = \$50 \\ 20 \text{ win } \$5 = +\$100 \\ \hline \$150 \end{array}$$

So \$1.50/game.

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### Expected Value:

The expected value of a random variable  $x$  is calculated by multiplying each value of  $x$  by its probability, and then adding up these products.

Simple Example: The probability of scoring  $x$  points in a carnival game is as follows. What is the expected number of points earned?

$x$		$p$	
1	x	.05	= .05
2	x	.10	= .20
3	x	.20	= .60
4	x	.50	= 2.00
5	x	.15	= .75
			<hr/>
			3.60

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### Carnival Game

1. For example, a carnival game is played where three coins are tossed and payoffs depend on how many heads are showing. The player gets \$1 for each head showing. Find the expected value of this game.

*x is the amt of money won.*

\$0	-	$\frac{1}{8}$	
\$1	-	$\frac{3}{8}$	
\$2	-	$\frac{3}{8}$	
\$3	-	$\frac{1}{8}$	

$$0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right) = \frac{12}{8} = \$1.50$$

2. If it costs \$2 to play this game, what is the expected net profit per game by the carnival?

$$\$2 - \$1.50 = \$.50$$

3. What price would make this a fair game, so that the players can expect to break even on average?

$$\$1.50$$

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### Playing the Lottery

In a state lottery, a player chooses four digits (0-9) in a specific order. (Numbers like 0051 and 0120 are possible). If the lotto selects four digits randomly and any player matches all four digits, they win \$6,000. If they match 3 digits, they win \$100. What is a fair cost to play this game?

$$P(\$6000) = \frac{1}{10^4} \text{ (match all 4)} \quad \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$$

$$P(\$100) = \frac{36}{10^4} \text{ (match any 3)} \quad \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{9}{10} = \frac{9}{10^4}$$

one option: (there are 3 others like this!)

$$\text{Expected Value} = (6000)\left(\frac{1}{10^4}\right) + 100\left(\frac{36}{10^4}\right)$$

$$= \$60 + \$36$$

$$= \$.96$$

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## Insurance Company

An insurance company insures an \$8000 car for an annual premium of \$740, with a \$500 deductible. If the company spends \$20 per year to service such a policy, the probability of total loss is 0.6%, and we assume that either a total loss or no loss will occur, what is the expected profit of the insurance company on such a policy?

$$\begin{aligned} & (-\$7500)(.006) - \$20(1.0) + 740(1.0) \\ & = \$675 \end{aligned}$$

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## Purchase Extended Warranty?

I recently purchased a \$99 gas grill from a retail store. The store offered to sell me an Extended Service policy for \$10, meaning they will repair the grill for free if any major problem develops during the first year.

Suppose the probabilities and costs of various repairs levels are listed below. Should I have purchased the Extended Service policy?

<u>Repair cost</u>		<u>Probability</u>	
<del>\$0-20</del>	*	0.04	→ .8
<del>\$20-40</del>		0.02	→ .8
<del>\$40-60</del>		0.01	→ .6
<del>\$60-99</del>		0.005	→ .45
			<hr/> \$2.65

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