

Sec. 13.3 - Measures of Dispersion (from last time)

HW #3, 7, 11, 15, 21, 23,
29-33 all, 46*, 47*, 52, 53

* Remind me to talk about these two on Monday!

We can start with a couple of homework questions.

1st Monthly Seminar on Teaching Mathematics

Tuesday, 10/7, at 4pm in Cowley 041

"A Discussion on Teaching for Understanding"

Andy Belter, UW-L Senior

Oct 2-8:12 AM

Homework Questions:

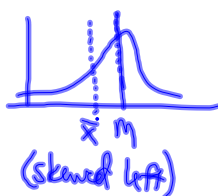
*46-47. The skewness coefficient is $k = 3(\text{mean} - \text{median})/(\text{st.dev})$

46. When is k positive? When is it negative?

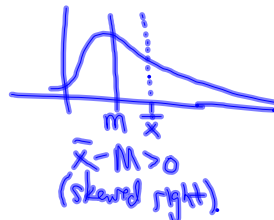
k is positive if and only if $(\text{mean} - \text{median}) > 0$.
That is, $\text{mean} > \text{median}$.



47. Why is the mean of a skewed distribution always farther out toward the tail than the median?



(skewed left)



$\bar{x} - m > 0$
(skewed right)

mean = $\frac{\text{sum of data}}{\text{number of data}}$, and

if an extreme value is present the sum will be influenced a lot.

Oct 6-9:47 AM

11)

Val	Freq	Deviation ²
9	3	$(3 \cdot 9)^2$
7	4	$(1 \cdot 9)^2$
5	7	$(6 \cdot 1)^2$
3	5	$(2 \cdot 1)^2$
1	2	$(4 \cdot 1)^2$
	21	

$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (n=21)$
 $\bar{x} = \frac{3(9) + 4(7) + 7(5) + 5(3) + 2(1)}{21}$
 $\bar{x} = 5.1$
 $\sum_{i=1}^{21} (x_i - \bar{x})^2 = 3(3 \cdot 9)^2 + 4(1 \cdot 9)^2 + \dots + 2(4 \cdot 1)^2$
 $= 115.81$
 $S = \sqrt{\frac{115.81}{20}} \approx 2.4$

9,9,9,7,7,7,...

Oct 6-10:08 AM

15) $1 - \frac{1}{K^2} = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$
 $K = \frac{3}{2}$

21) $\bar{x} = 70$
 $S = 8$

Chan: 54, 86
 Chebyshev's:
 $1 - \frac{1}{2^2} = 75\%$

23)

$K=4$ here.
 $1 - \frac{1}{4^2} = 1 - \frac{1}{16} = \frac{15}{16}$

st. dev. = 8
 38, 102

Oct 6-11:14 AM

31) 80, 105, 120, 175, 185, 190
 215, 210, 215, 300, 320, 325

(29) $\bar{x} = 202.50$
 (30) $s = 80.30$

range w/in ± 1 st. dev. is } 6 of 12,
 (202.50 - 80.30) to (202.50 + 80.30) } or 50%
 \uparrow 122.20 to \uparrow 280.80

33) $1 - \frac{1}{2^2} = \frac{3}{4}$
 at least $\frac{3}{4}$ of (12) = 9 bonuses w/in ± 2 .

Oct 6-11:19 AM

Reaction Time Data:

9:55am

95% within 2 st.dev

11am

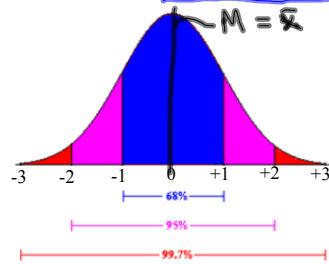
96% within 2 st.dev

(Chebyshev's Rule says at least $1 - 1/4 = 75\%$ should be within 2 st.dev of the mean value.)

Oct 6-9:00 AM

Measures of spread, continued:

Empirical Rule gives more precise information about a data set than the Chebyshev's Theorem, however it only applies to a data set that is normally distributed (i.e., bell curves).



68% of the observations lie within one standard deviation of the mean.

95% of the observations lie within two standard deviations of the mean.

99.7% of the observations lie within three standard deviations of the mean.

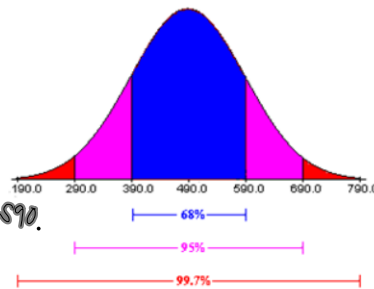
Oct 2-8:40 AM

Bell Curve Example

The scores for all high school seniors taking the verbal section of the Scholastic Aptitude Test (SAT) in a particular year had a mean of 490 and a standard deviation of 100. The distribution of SAT scores is bell-shaped.

1. What percentage of seniors scored between 390 and 590 on this SAT test?

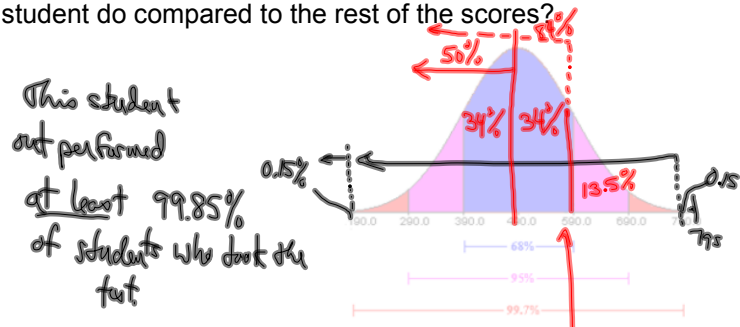
± 1 st. dev.
Use the empirical rule,
68% of students
scored between 390 and 590.



Oct 2-8:40 AM

Recall: The scores had a mean of 490 and a standard deviation of 100.

2. One student scored 795 on this test. How did this student do compared to the rest of the scores?



3. A rather exclusive university only admits students who were among the highest 16% of the scores on this test.

What score would a student need on this test to be qualified for admittance to this university?

590.

Oct 2-8:40 AM

Summary:

The range is a rough measure of spread. ($R = \max - \min$)

(?) Is it a resistant statistic?

The standard deviation (and variance) are two other ways to measure spread.

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

(?) Is st.dev. a resistant statistic?

Standard deviation can be interpreted as a sort of "average" deviation from the mean.

(a) Chebyshev's Theorem and (b) the Empirical Rule are two different tools for interpreting the st.dev.

(?) Which is more accurate?

(?) When can you use them?

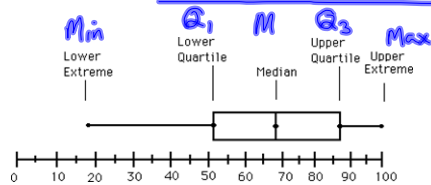
Oct 2-9:24 AM

Section 13.4
Box plots and Measures of Position

HW: 18, 20, 29, 41

Oct 6-9:32 AM

Box Plot & 5 Number Summary



- The "box" in the box plot contains, the middle half of the data points.
- The median divides the data into two halves. To divide the data into **quarters**, you then find the medians of these two halves. These values are called Q₁ for the lower half and Q₃ for the upper half.
- These five values (Min, Q₁, Q₂ = Median, Q₃, Max) are referred to as the **five number summary**.

Oct 6-9:32 AM

Construct a box plot.

The depth of snow at a ski resort are collected every year for 12 years on the 1st of February. All data is in centimeters and arranged in numerical order...

40, 45, 55, Q_1 60, 65, 65, m 70, 75, 75, Q_3 80, 85, 90

$$\text{Min} = 40$$

$$Q_1 = 57.5$$

$$\text{Med} = 67.5$$

$$Q_3 = 77.5$$

$$\text{Max} = 90$$

Oct 6-9:32 AM