Sec. 13.5 - The Normal Table

HW: 7-10, 11-14 <-- quiz worthy

Can wait 'til tomorrow --> 15-25 odd, 37-40, 45, 47, 56-60

In a symmetric distribution, the z-score can be used to find the percentage of data points above, below or in between data values.

E.g. Empirical Rule for the Normal Distribution

Oct 8-9:17 AM

Quick Review of z-scores:

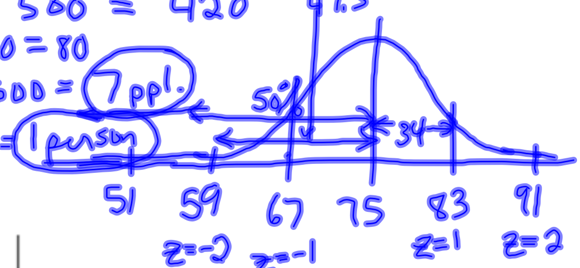
If 500 people took a placement test, which has a mean of 75 and a standard deviation of 8, how many people would get the following scores? (assuming the scores are normally distributed)

a) less than 83? 84% of 500 = 420

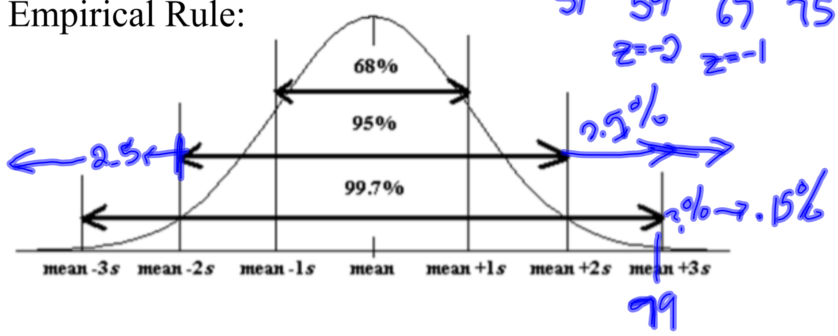
b) higher than 83? 500 − 420 = 80

c) less than 59? 2.5% of 500 = 7 ppl.

d) at least 99? .15% of 500 = 1 person



Empirical Rule:



Quiz Thursday on 13.1 - 13.4, plus 13.5's empirical rule and z-scores.

Oct 8-8:56 AM

Z-score as a "standardized" score:

The *z*-score gives us a way to compare raw scores from different distributions.

EX: The scores for all high school seniors taking the math section of the SAT in a particular year had a mean of 490 and a standard deviation of 100.

In the same year, the scores for all high school seniors taking the math section of the ACT was 21 with a standard deviation of 5.

On the math portions of these standardized tests, you scored a 600 on the SAT and your friend scored a 28 on the ACT. Which score is relatively better?

SAT: $z = \frac{\text{score} - \text{mean}}{\text{st.dev}} = \frac{x - \bar{x}}{s} = \frac{600-490}{100} = 1.1$

ACT: $z = \frac{28-21}{5} = \frac{7}{5} = 1.4$ } ACT score is (relatively speaking) better than the SAT score.

Oct 7-8:17 AM

Take it a step further:

Melissa and Mike took a test, and their scores came back as follows:

	test	z-score
Melissa:	78	1.6
Mike:	68	-0.4

$\bar{x} = 70$

$1.6 = \frac{78-70}{s} \overset{(*)}{\Rightarrow} s = 5.$

What must the mean and standard deviation on the test have been?
(this should help with one of the homework problems from 13.4)

Melissa: $1.6 = \frac{78-\bar{x}}{s} \xrightarrow{(*)} s = \frac{78-\bar{x}}{1.6}$ } #34

Mike: $-0.4 = \frac{68-\bar{x}}{s} \xrightarrow{(*)} s = \frac{68-\bar{x}}{-0.4}$

So $\frac{78-\bar{x}}{1.6} = \frac{68-\bar{x}}{-0.4}$ (cross multiply)

$-0.4(78-\bar{x}) = 1.6(68-\bar{x})$

$\Rightarrow -31.2 + .4\bar{x} = 108.8 - 1.6\bar{x}$

$(*)$ $+31.2 \quad +1.6\bar{x} \quad +31.2 \quad +1.6\bar{x}$

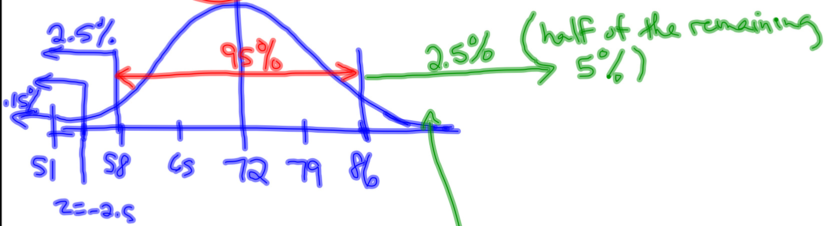
$\Rightarrow \boxed{\bar{x} = 70}$

Oct 7-9:28 AM

Practice:

The final exam scores in Bio 103 has a *normal distribution* with a mean of 72 and a standard deviation of 7.

a) What percent of all Bio 103 students scored between 58 and 86? 95% (btw'n ± 2 st. dev. of the mean)



b) If 300 students took the exam, find the number of students that scored above 86.

$2.5\% \times 300 = 7.5.$ So ≈ 7 people

c) What Bio 103 score corresponds to a z-score of -2.5?

$$z = \frac{x - \bar{x}}{s}$$

$$-2.5 = \frac{x - 72}{7}$$

$$\Rightarrow x = 54.5$$

Oct 7-8:17 AM

The final exam scores in Bio 103 has a *normal distribution* with a mean of 72 and a standard deviation of 7.

d) Estimate the percentage of students who scored below 60 on the Bio 103 exam.

$$z = \frac{60-72}{7} = \frac{-12}{7} = -1\tfrac{5}{7} \approx -1.71$$

Because z = -1.71 is not an integer value, our Empirical Rule does not help. Instead, this question is best answered with a normal probability table like Table 10 in your book... here's how:

Look up z=1.71 in Table 10. It gives an area of A = 0.456.

Now we calculate 0.500 - 0.456 = 0.044. Thus, 4.4% of the scores are *above* z = 1.71.

At this point you should ask, "but why did you find the scores *above* z = *positive* 1.71?"

Here's why: Table 10 does not include any negative z-scores.

So even though we want the % of scores *below* negative 1.71, we have to look up z = positive 1.71 and use the symmetry in the table to see that the same number of scores (4.4%) will be above z=1.71 as below z=-1.71. So our answer is 4.4% of students scored below 60 on the Bio 103 exam.

More on this tomorrow!

Oct 8-9:14 AM

Meaning of the Normal Curve Areas:

In the standard normal curve (Table 10), the following three quantities are equivalent:

1. Percentage (of total items that lie in an interval)
2. Probability (of a randomly chosen item lying in an interval)
3. Area (under the normal curve along an interval)

The normal table (Table 10, p. 766) gives us the areas under the "standard" normal curve.

Note: The *standard* normal curve has a mean of 0 and a standard deviation of 1.

When we compute the z-score for an item, we are converting it so we can compare it to other scores on the standard normal curve.

Oct 8-9:44 AM

Tomorrow:

More practice with the standard normal curve and Table 10.

For the quiz, focus on the 13.5 homework dealing with the Empirical Rule (#7-14) along with the homework from preceding sections.

We'll use the rest of class time today for a study session, as needed.

Oct 8-9:48 AM