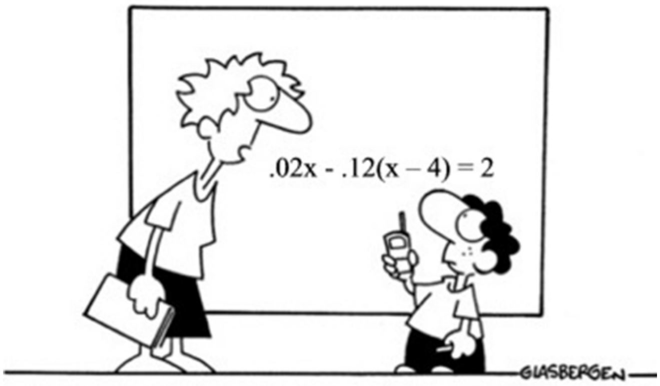
Unit 3 - Algebra (Ch. 7 and 8)

HW: Sec. 7.1 #7, 19, 23, 25, 31, 35, 39, 45, 50, 51, 53, 57, 67, 71




"You have to solve this problem by yourself. You can't call tech support."

Oct 20-8:35 AM

Research on the Learning of Algebra:

- We all have a vested interest in teaching & learning of algebra.

- Students who struggle in algebra also struggle in future math classes.

- Dr. Kosiak and I often work with teachers and write articles related to the teaching and learning of algebra.

Therefore...

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Oct 20-8:37 AM

Number tricks:

- Think of a Secret Number
- Add 7
- Multiply by 2
- Subtract 4
- Divide by 2
- Subtract you Secret Number
- Your result is …

**5**

Why does it work?

$$\frac{2(x+7)-4}{2} - x$$

$$= \frac{2x+14-4}{2} - x$$

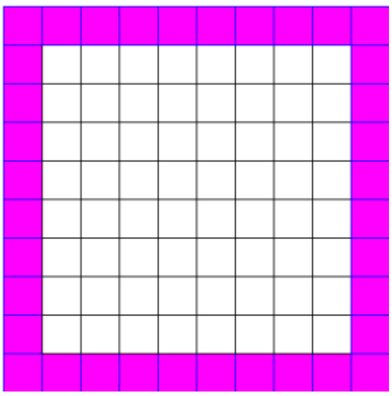
$$= \frac{2x+10}{2} - x = \frac{\cancel{2}(x+5)}{\cancel{2}} - x$$

$$= x+5-x = 5$$

Oct 20-8:42 AM

# Border Problem

Below is a 10 by 10 grid. Without writing, without talking, and without counting one by one, how many unit squares are in the border (colored portion).



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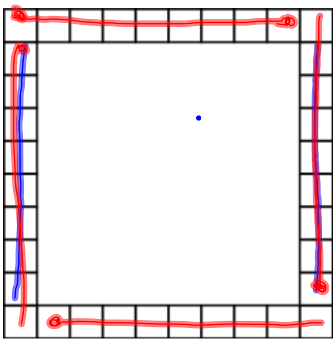

What if it was a 6x6 grid?

# Border Problem

| 10 x 10 <br> 36 | 6 x 6 <br> 20 | n x n |
|---|---|---|
| $2(10)+2(8)$ | $2(6)+2(4)$ | $2n+2(n-2)$ |
| $4(10)-4$ | $4(6)-4$ | $4n-4$ |
| $4(8)+4$ | $4(4)+4$ | $4(n-2)+4$ |
| $10^2-8^2$ | $6^2-4^2$ | $n^2-(n-2)^2$ |

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# Border Problem

| 10 x 10 | 6 x 6 | n x n |
|---|---|---|
| 2 x 10 + 2 x 8 | 2 x 6 + 2 x 4 | 2n + 2(n – 2) |
| 4 x 10 – 4 | 4 x 6 – 4 | 4n – 4 |
| $10^2 - 8^2$ | $6^2 - 4^2$ | $n^2 - (n-2)^2$ |
| 4 x 9 | 4 x 5 | 4(n – 1) |

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Terminology of Algebra (p. 315):

- Algebraic Expression    eg: $(2x+1)^2$
  * Involves operations of [+, -, x, ÷, powers, roots]

- Equation is a statement that two expressions are equal.
  * Linear equation: can be written in the form "ax + b = c"

- Solution
  * A number that makes an equation true.
    e.g. x - 3 = 5 (find a solution)    $x=8$

- Solution Set
  * The set of all solutions to an equation.
    e.g. x - 3 = 5 (what is the solution set?)    $\{8\}$

- Equivalent equations
  * Two equations are equivalent if they have the same solution set.
    e.g. 8x + 1 = 17 and 8x = 16 are equivalent equations.

(solution set is $\{2\}$ in each case.)

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Key Principles of Algebra:

- Distributive Property:
  a(b + c) = ab + ac

- Combining Like Terms:
  **(Why does it work?)** Combining like terms is just an application of the distributive property!
  e.g. (2x + 3x) - 7 = 8

$\Rightarrow x(2+3)-7=8$

$\Rightarrow 5x-7=8$

etc.

- Addition Property of Equality: For any a, b, and c,
  a = b and a + c = b + c are equivalent equations.

- Multiplication Property of Equality: If c ≠ 0 then
  a = b and ac = bc are equivalent equations.

Caution:

1) $0\cdot(5x+2)=(7)\cdot 0 \Rightarrow 0=0$

2) $(x-1)\dfrac{2(x+1)}{x-1}=4(x-1)$ → (ok unless $x=1$)

Oct 20-9:10 AM

Solving a Linear Equation in One Variable:
1. Clear Fractions
2. Simplify Each Side Separately
3. Isolate the Variable Terms on One Side
4. Transform so the Coefficient of the Variable is 1
5. Check

Examples:
Solve: $12 - 2x = 4$

$+(-12) \quad +(-12)$ (additive property)

3. $12 + (-12) - 2x = 4 + -12$

$-\frac{1}{2}(-2x) = (-8)(-\frac{1}{2})$ (mult. property)

4. $x = 4$

5. Replace x with 4:

$12 - 2(4) \stackrel{?}{=} 4$

$12 - 8 \stackrel{?}{=} 4$

$4 = 4$ ✓

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Solving a Linear Equation in One Variable:
1. Clear Fractions
2. Simplify Each Side Separately
3. Isolate the Variable Terms on One Side
4. Transform so the Coefficient of the Variable is 1
5. Check

check: $-2 + 12 \stackrel{?}{=} 10$ ✓

$2(4-5) + 3(4) \stackrel{?}{=} 4 + 6$

Solve: $2(k - 5) + 3k = k + 6$

$\Rightarrow 2k - 10 + 3k = k + 6$

$\Rightarrow 5k - 10 = k + 6$
$-k \quad -k$

$\Rightarrow 4k - 10 = 6$
$+10 \quad +10$

$\Rightarrow \frac{4k}{4} = \frac{16}{4}$

$\Rightarrow k = 4$

Solve: $10\left(\frac{4x+5}{5} - \frac{2x-3}{2} = 2\right)10$

Step 1: clear fractions: x by 10

Step 2: $2(4x+5) - 5(2x-3) = 20$

$8x + 10 - 10x + 15 = 20$

$-2x + 25 = 20$
$-25 \quad -25$

Step 3: $\frac{-2x}{-2} = \frac{-5}{-2}$

Step 4: $x = \frac{5}{2}$

Step 5: Check:

Check:
$\left(\frac{10+5}{5}\right) - \frac{(5-3)}{2}$

$3 - 1 \stackrel{?}{=} 2$ ✓

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Examples:

Solve:

$100(.02x - .12(x-4)) = 2(100)$

$2x - 12(x-4) = 200$

$2x - 12x + 48 = 200$

$-\frac{1}{10}(-10x) = 152\left(-\frac{1}{10}\right)$

$x = -15.2$

Solve:

-6k + 2k -11 = 4 – 2(2k – 3)

$-4k - 11 = 4 - 4k + 6$

$-4k - 11 = 10 - 4k$

$+4k \qquad\qquad +4k$

$-11 = 10 \longrightarrow$ solution set is $\{\}$!

Solve:

4[6 – (1 + 2m)] + 10m = 2(10 – 3m) + 8m

(HW: solve)

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Types of Equations:

If an equation has...

... a finite number of solutions, we say it is a **conditional equation.**

... an infinite number of solutions, we say it is **an identity.**

... no solutions, we say it is **a contradiction.**

Examples: Classify these equations by their solution types.

Solve: $\frac{4x+5}{5} - \frac{2x-3}{2} = 2$

Solution Set: {5/2}

Solve: -6k + 2k -11 = 4 – 2(2k – 3)

Solution Set: {}, or Ø

Solve: 4[6 – (1 + 2m)] + 10m = 2(10 – 3m) + 8m

Solution Set: {All real numbers}

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