7.3 - Ratio*, Proportion*, and Variation
HW: 19, 21, 23, 25, 29, 31, 33,
47, 55-57, 59, 62, 66, 67, 70, 72

* Our focus in Mth126 will be on direct and inverse variations because of their central role in early algebraic thinking.

* Ratio and Proportion are also covered extensively in the Mth125 curriculum.

First, we'll take some homework questions.

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Homework questions, Sec. 7.2.

37) 12 L, 12% alcohol
another @ 20%
want 14% alcohol solution

Consider L of alcohol:

$$(12L)(.12) + x(.20) = (12+x)(.14) \checkmark$$

Consider the raw quantity of the item of interest. (eg; told about % alcohol, but the equation is about Liters of alcohol)

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45) $i = p \cdot r$

Let x = amt of money @ 4.5%.

$(2x - 1000)$ = amt @ 3%

$$x(.045) + (2x - 1000)(.03) = \$1020$$

etc.

$x = 10{,}000$ @ 4.5.

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47) 29,000 @ 5%

x @ 2% (Let x be amt invested @ 2%)

want to get: return of 3%

$i = p \cdot r$

$$(29000)(.05) + x(.02) = (29000 + x)(.03)$$ ✓

... $x = \$58{,}000$

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A **ratio** is a quotient of two quantities.

e.g $\frac{a}{b}$

A **proportion** is an equation asserting that two ratios are equivalent.

e.g. $\frac{3}{15} = \frac{x}{10} \Rightarrow 30 = 15x \Rightarrow x = 2$

Proportions can be solved using **cross multiplication**. That is, as long as none of the denominators are zero:

$$\frac{a}{b} = \frac{c}{d} \text{ is equivalent to } ad = bc$$

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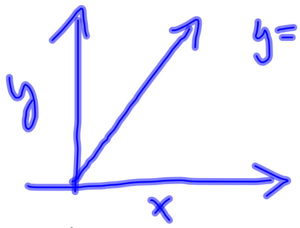
Direct Variation

We say that ***y* varies directly as *x*** or that ***y* is directly proportional to *x*** if there exists some constant $k \neq 0$ such that

$$\boxed{y = kx}$$

or, equivalently,

$$\boxed{\frac{y}{x} = k}$$



$y = kx + b$, $b = 0$

We say that k is the **constant of variation** or the **constant of proportionality.**

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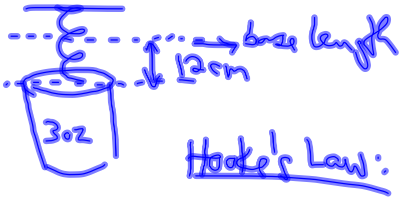
Example (Hooke's Law):

For any elastic spring (rubber band, etc.), the distance the spring stretches is directly proportional to the force applied.

Suppose a light spring stretches 12 cm when a force of 3 oz is applied. How much force is needed to stretch the spring to 20 cm?

$d = 20$



Hooke's Law: $d = kf$

Use initial condition to find k.

$\therefore \; 12 = k(3)$

so $k = 4$.

Now: $d = 4f$ is the equation for this spring.

$\therefore \; 20 = 4f \Rightarrow f = 5$ oz.

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Skittles Crane Activity:

get 6 or 7 data points

1. Use a light rubber band to suspend a plastic cup from a pencil.
2. Record the base (unstretched) length of the rubber band.
3. Add a fixed quantity of Skittles (or similar) and measure the total stretch.
4. Repeat and observe how the length of the rubber band changes with the addition of more weight.
   a) Find the constant of proportionality for your rubber band.
   b) Predict the length of the rubber band if you were to add 18 additional Skittles (or similar).

x	y
20 pennies	1.3 cm
+18 pennies	? cm?

$y = .073x$

$y = .073(38) \approx 2.774$

vs. $y = .073(20) \approx 1.46$

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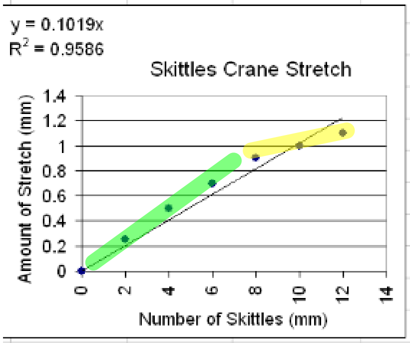
Results:

Team 2:



x	y	y/x
Skittles	Stretch	k
0	0	#DIV/0!
2	0.25	0.13
4	0.5	0.13
6	0.7	0.12
8	0.9	0.11
10	1	0.10
12	1.1	0.09
	avg	**0.11**

(this average gives a good estimate for the constant of proportionality)

Notice that we can see two different slopes emerging in this data. Some rubber bands reach a "yield point" beyond which their stretching behavior changes.

Once the yield point is exceeded, the rubber band may not return to its original shape.

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Capture / Recapture:

Biologists use algebra to estimate numbers of organisms in an environment using a technique called capture / recapture.

For instance, a biologist might catch and tag 140 fish in a given lake at the beginning of the month. Two weeks later, the biologist returns and catches 185 fish.

If 12 of the 185 fish had been previously tagged, estimate how many fish (to the nearest hundred) are in the lake altogether.

2nd Sample:

$$\frac{12 \text{ tagged}}{185 \text{ total}} = \frac{140 \text{ tagged}}{x \text{ total}}$$

$x \approx 2200$ fish.

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