Today's Agenda:

- More examples from 7.3 (Proportional Variation)
- Start 7.4 (Linear Inequalities)

- Looking Ahead:
  * Quiz Monday (NOT Thursday)
  * Problem Set due Thursday
  * Tomorrow: Sec. 7.4 (Linear Inequalities) and start Sec. 7.5 (Exponents)

7.3 HW (from last time):
19, 21, 23, 25, 29, 31, 33,
47, 55-57, 59, 62, 66, 67, 70, 72

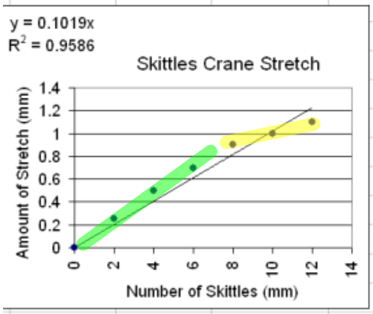
Oct 29-8:38 AM

Skittles Crane Follow-up:

On Monday, we found a nice (Hooke-ian) proportional relationship between the stretch of our rubber bands and the weight attached.

Two things to note:
1. One group had a data set that showed a possible "yield point."
2. (Ahead a few slides)

Team 2:



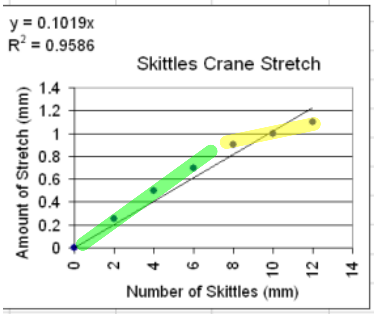
x	y	y/x
Skittles	Stretch	k
0	0	#DIV/0!
2	0.25	0.13
4	0.5	0.13
6	0.7	0.12
8	0.9	0.11
10	1	0.10
12	1.1	0.09
	avg	**0.11**

(this average gives a good estimate for the constant of proportionality)

Oct 29-8:51 AM

Yield Point:

Team 2:



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Notice that we can see two different slopes emerging in this data. Some rubber bands reach a "yield point" beyond which their stretching behavior changes.

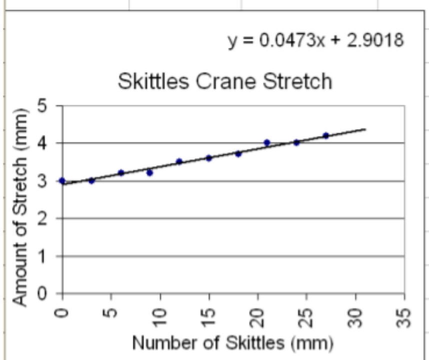
Once the yield point is exceeded, the rubber band may not return to its original shape. (We also can no longer say it follows a proportional variation!)

Oct 29-9:03 AM

2. The Skittles Crane can be used to illustrate *non*-proportional variation as well by considering <u>total length</u> (instead of <u>stretch</u>) as a function of the weight attached.

How can you tell this data follows a *non*-proportional variation?

Team 4:



Skittles	Length
0	3
3	3
6	3.2
9	3.2
12	3.5
15	3.6
18	3.7
21	4
24	4
27	4.2
31	4.5

- Doesn't go through (0,0).

- Best fit line is not of form y = kx.

- Doubling the weight does not double the length.

Oct 29-8:54 AM

Example: Weight and Pressure

The weight w that a given tire supports varies directly with the tire's "footprint" A (the rectangular area where the tire contacts the ground).

We can use the air pressure p of the tire as the constant of proportionality, so that $\boxed{w = pA}$.

Rewriting that equation, we have $\boxed{w / A = p}$. So the ratio of weight to area is constant (if we don't let any air out of the tire!)

1. If a given tire's pressure is 32 psi and the tire's footprint is 30 square inches, how much weight is that tire supporting?

$w = pA \Rightarrow w = (32)\frac{\text{lbs}}{\text{sq in}}(30)\text{ sq in} = 960 \text{ lbs.}$

2. After filling the gas tank, the tire's footprint 31 square inches. How much additional weight is the tire supporting now?

$w = pA$

$w = (32)(31) = 992 \text{ lbs.}$

(that's 32 more lbs.)

-OR- $\frac{960}{30} = \frac{w_{new}}{31}$

$\frac{w}{A}$ $\frac{w}{A}$

again, $w_{new} = 992$.

Oct 29-9:02 AM

How to weigh a car...

...using two pieces of paper and a pressure gauge!

Oct 29-9:07 AM

Solving a variation problem:

1. Write the variation equation. (eg: $y = kx$, or $y = \frac{k}{x^2}$)
2. Substitute the initial values and solve for the constant k.
3. Rewrite the variation equation using the value of k from step 2.
4. Substitute the remaining values, solve, and answer the question.

Example (skydiver):

The distance a body falls from rest varies directly as the *square* of the time it falls (disregarding air resistance). If a skydiver falls 64 feet in 2 seconds, how far will she fall in 8 seconds?

1. $d = kt^2$
2. $\frac{64}{4} = \frac{k(2^2)}{4} \Rightarrow k = 16$
3. $d = 16t^2$
4. $d = 16(8^2)$

   $d = 1024$ ft

- fell for 4x as many seconds
- fell 16x as far.

We say a **y varies inversely as *x*** if there exists a real number k such that $xy = k$ (or, equivalently, $y = k / x$).

Similarly, **y varies inversely as x^n** if $y = \frac{k}{x^n}$ for some constant k.

Comlete the following table to explore the relationship where y varies inversely as x^2. (First, find the constant of proportionality using the given information.)

x	y
3	256
6	64
$\boxed{12}$	$\boxed{16}$
24	4
30	2.56

÷4 ÷2 ×2 ×4² ×4 ÷4

initial condition

1. $y = \frac{k}{x^2}$
2. $\Rightarrow 16 = \frac{k}{(12)^2}$

   $\Rightarrow 16(12^2) = k = 2304$
3. $y = \frac{2304}{x^2}$

Example: (Music at a Concert)

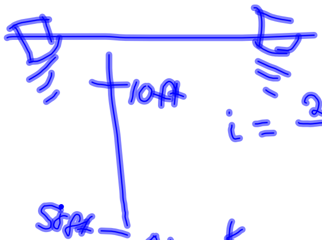
Formulas for converting between intensity & decibels:
$d = 120+10*\log(i)$ $\quad$ $i = 10^{[d/10)-12]}$

The intensity of sound varies inversely as the square of the distance from the source.

So $i = \dfrac{K}{d^2}$

At a concert, you and your friends start out 50 feet from the stage where the speakers are. At this distance, the music's intensity is about 0.1 watts per square meter (W/m^2), or about 110 decibels.

Later, you and your friends work your way to about 10 feet from the stage. How many times more intense is the music at this distance?



$i = 0.1$ $\quad$ $0.1 = \dfrac{k}{50^2} \Rightarrow K = 250$

$i = \dfrac{250}{10^2}$ $\quad$ 2.5

.1 → 2.5 is an increase of 25 times in intensity.

Key Points from Sec. 7.3:

* In upper elementary / middle school math, there is a big focus on linear relationships, including proportional variation.

From *PSSM*: In Grades 6-8 all students should --

Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Use graphs to analyze the nature of changes in quantities in linear relationships.

* As a teacher, you should be able to quickly identify situations that are linear / proportional and know how to flexibly represent them symbolically, graphically, and using tables.

Sec. 7.4 - Linear Inequalities

A linear equation is one that can be written as ax+b = c.

A linear inequality can be written as ax + b < c.
(or replace < with >, ≤, or ≥.)

Addition Property: a < b is equivalent to a + c < b + c.

Two equations (or inequalities) are equivalent if they have the same solution sets.

Oct 29-9:26 AM

Example: Solve the following linear inequalities. Express your answer in set notation, as an interval, and on a number line.

1. $x - 7 < 15 - 2x$

$\quad +7 \quad\quad +7$

$x - \cancel{7} + \cancel{7} < 15 + 7 - 2x$

$+2x + x < 22 - \cancel{2x} + \cancel{2x}$

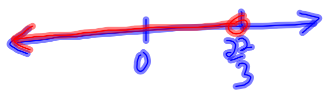
$\frac{1}{3} \cdot (3x) < (22) \cdot \frac{1}{3}$

$x < \frac{22}{3}$ (mult. property)

"such that"

$\{x \mid x < \frac{22}{3}\}$

$(-\infty, \frac{22}{3})$



2. $5 - x < 8$

$-5 \quad\quad -5$

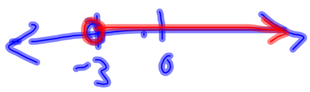
$-x < 3$
$+x \quad\;\; +x$

$0 < 3 + x$
$-3 \quad -3$

$-3 < x$

$(-1)(-x) < 3(-1)$

$x > -3$



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