Thursday, 10/30/08

Sec. 7.4 - Linear Inequalities
#11, 15, 19, 23, 27, 29, 35, 39, 45, 47, 57, 59, 61, 64,
plus additional problems on absolute value (handout).

Problem Set 3 Due
Quiz Monday on 7.1 through 7.3

Oct 30-8:19 AM

Multiplication Property:

a < b is equivalent to ac < bc if c > 0.

a < b is equivalent to ac > bc if c < 0.

change the sense of < to >

x = 0 does not check!

$0 - 12 \overset{?}{\geq} 6 - 0$

Example:

1. Solve $-3x - 12 \geq 6 - x$.

$$\begin{aligned} -3x - 12 &\geq 6 - x \\ +12 \quad & \quad +12 \\ -3x &\geq 18 - x \\ +x \quad & \quad +x \\ -2x &\geq 18 \end{aligned}$$

Incorrect: $\frac{-2x}{-2} \geq \frac{18}{-2} \Rightarrow x \geq -9.$

$$\frac{-2x}{-2} \leq \frac{18}{-2}$$

$$x \leq -9$$

check if you wish by choosing an $x \leq -9$ and evaluating in the original.

Oct 30-8:35 AM

Interpreting the Answer: What types of solutions are possible when solving linear inequalities? Here are a few examples. Write as intervals and graph the solution set.

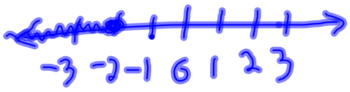
$x < 3$

$(-\infty, 3)$

$x > 5$

HW

$x \leq -2$



$(-\infty, -2]$

$x \geq 4$

HW

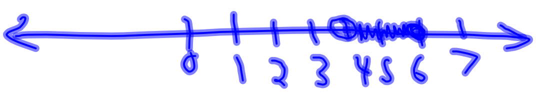
Oct 30-8:37 AM

Interpreting the Answer:

For compound inequalities, we can get other types of solutions, such as these examples. Write them as intervals and graph the solution set.

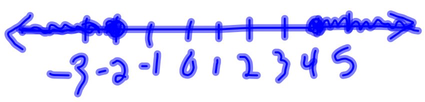
$4 < x \leq 6$

$4 < x$ and $x \leq 6$



$(4, 6]$

$x \geq 4$ or $x \leq -2$



$(-\infty, -2] \cup [4, \infty)$

Oct 30-9:10 AM

Conjunctions:

The notation 3 < 4 < 7 means (3 < 4 *and* 4 < 7). It is also interpreted as "4 is between 3 and 7."

Notice that if we add 6 to each part, we obtain another true inequality:

$$3+6 < 4+6 < 7+6$$
$$\Rightarrow \quad 9 < 10 < 13$$

Likewise, if we multiply 3 < 4 < 7 through by 2, we obtain another true inequality:

$$2(3) < 2(4) < 2(7)$$
$$\Rightarrow \quad 6 < 8 < 14$$

What if we multiply through by -3?

$3 < 4 < 7$

$\times -3$: $-9 \not< -12 \not< -21$

$-9 > -12 > -21$

Remember to change the sense of the inequality whenever you multiply through by a negative number.

Conjunction Examples:

Solve the following inequalities and graph the solution sets.

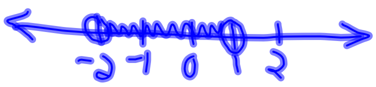
A) $-1 < 2x + 3 < 5$

$-3 \quad -3 \quad -3$

$-4 < 2x < 2$

$-4\left(\frac{1}{2}\right) < \frac{1}{2}(2x) < \frac{1}{2}(2)$

$-2 < x < 1$



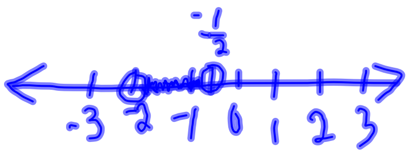
B) $5 < 3 - 4x < 11$

$-3 \quad -3 \quad -3$

$2 < -4x < 8$

$-\frac{1}{4}(2) > -\frac{1}{4}(-4x) > -\frac{1}{4}(8)$

$-\frac{1}{2} > x > -2$



Absolute Values and Inequalities:

|x| represents a number's distance from zero on the number line. ✓

The formal definition is:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$|-5| = -(-5) = 5.$

Interpret as a distance:

A) |2x + 7| > 5 — (2x+7) is more than 5 units away from 0.

B) |2x + 7| < 8 — (2x+7) is less than 8 units away from 0.

C) 2 < |2x + 7| < 9 — (2x+7) is between 2 and 9 units away from 0.

Oct 30-9:09 AM

Absolute Values and Inequalities:

A) $|2x - 3| < 5$ — (2x−3) is less than 5 units from 0, so it's between −5 and 5.

$$-5 < (2x-3) < 5$$
$$\quad +3 \qquad\quad +3 \quad +3$$
$$-2 < 2x < 8$$
$$\frac{-2}{2} < \frac{2x}{2} < \frac{8}{2}$$

$-1 < x < 4$, so the interval $(-1,4)$ is the solution to $|2x-3| < 5$.

B) $|4 - x| > 3$

"the value of (4−x) is more than 3 units away from 0."

So either

$4 - x < -3$, or $4 - x > 3$

$\quad -4 \quad\ -4 \qquad\qquad +x \quad +x$

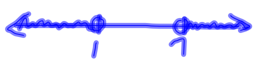
$\Rightarrow -x < -7$ or $4 > 3 + x$

$(-1)(-x) > (-1)(-7) \qquad -3 \quad -3$

$x > 7$ or $1 > x$

$(7,\infty) \cup (-\infty,1)$

i.e. $(-\infty,1) \cup (7,\infty)$



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So either

$4-x < -3,$ or $4-x > 3$

$-4 \quad -4 \qquad +x \quad +x$

Can we write this as:

$-3 > 4-x > 3$?

No: this would imply that $-3 > 3$.

what if we try:

$3 > 4-x > -3$

$-1 > -x > -7$

$1 < x < 7$.

Nope: wrong set!

$7 < x < 1$

Oct 30-11:41 AM

More Practice:

A) $-3 \leq \frac{1-4m}{3} < 2$

$\frac{10}{4} \geq m > -\frac{5}{4}$

B) $\frac{1}{5}(k-2) - \frac{3}{4}(2k-1) \leq 3$

$k \geq -\frac{53}{26}$

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Oct 30-9:17 AM