Sec. 7.6 - Polynomials and Factoring - Day 2

HW #5, 18, 19, 39*, 43*, 45*, 46, [47-99 e.o.o.], 100

* Also use the CFQ and Tic-Tac-Toe Methods on these

* Return quizzes & discuss.
* Finish up Sec. 7.6 on Polynomials and Factoring

Looking ahead: Next quiz will be Tuesday 11/11.

Nov 5-8:13 AM

Factoring Polynomials: A few special cases.

✔ Difference of squares:

$a^2 - b^2 = (a + b)(a - b)$

Ex: Factor completely: $288 - 18x^2$ $\rightarrow 18(16-x^2)$

$= 2(144-9x^2)$

$= 2(12^2-(3x)^2)$

$= 2(12+3x)(12-3x) = 2\cdot 3(4+x)\cdot 3(4-x)$

$= 18(4+x)(4-x)$

✔ Perfect Square Trinomials:

$a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

$a^2 - 2ab + b^2 = (a-b)^2$

Example: Fill in the blank in the polynomial below so that it is a perfect square trinomial. Then factor it completely.

$x^2 + \overset{12a}{12x} + \underline{36} = (x+6)^2$

$\downarrow$
a^2

so $a = x$

Nov 5-8:22 AM

Factoring Polynomials: A few special cases.

✗ Sum of squares:
$a^2 + b^2 = ????$ (prime!)

✓ Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(Verify)

$$\begin{array}{rr} & a^2 - ab + b^2 \\ \times & a + b \\ \hline & \cancel{a^2b} - \cancel{ab^2} + b^3 \\ + \; a^3 & \cancel{-a^2b} + \cancel{ab^2} \\ \hline a^3 & + \quad b^3 \end{array} \quad \checkmark$$

✓ Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Factor completely:
$x^6 - 125$

$(x^2)^3 - 5^3 = (x^2 - 5)(x^4 + 5x^2 + 25)$

Nov 5-8:27 AM

More complex cases: when the lead coefficient is not 1.

Factor the following trinomials:

a) $6x^2 + 11x + 4$

$(3x + 4)(2x + 1)$

b) $12x^2 - 11x - 36$

$(\quad + \quad)(\quad - \quad)$

$-$ $+$

Trial and error is too slow!
What we need here is some strategy.

Nov 4-9:08 AM

Factoring by Grouping: (Why factors of ac?)

Given $ax^2 + bx + c$….

Find two factors of ac that add to b.

$ac = -36$ $\quad 1(-36) \quad (-1)(36) \quad (-2)(18) \quad (-3)(12) \quad (-4)(9) \quad (-6)(6)$ etc.

Questions:

a) Does $3x^2 + 7x – 12$ factor using integer coefficients?? NO

b) Given $3x^2 + bx – 12$, what can we replace b by so the trinomial factors with integer coefficients?

b could be:
$-35,\ 35,\ 5,\ 16$, etc.

Example of factoring by grouping:

$3x^2 + 5x - 12$

$= 3x^2 + (9x - 4x) - 12$ what if these are reversed?

$= (3x^2 + 9x) + (-4x - 12)$

$= 3x(x+3) + -4(x+3)$

$= (3x-4)(x+3)$

Nov 4-9:12 AM

Other Methods of Factoring:

1. Common Factor Quotient (CFQ) Method
2. Tic-Tac-Toe Method

First, we'll illustrate the CFQ method of factoring.
(Full description on next slide).

Factor the following trinomial using the CFQ Method:

$6x^2 + 11x + 4$ $\quad 6\times4=24$, $8\cdot3$

$\dfrac{(6x+3)(6x+8)}{6}$

$\dfrac{\cancel{3}(2x+1)\cancel{2}(x+4)}{\cancel{6}}$

$3x^2 + 5x - 12$ $\quad$ Need $3(-12) = -36$, $9(-4)$

$= \dfrac{(3x+9)(3x-4)}{3}$

$= \dfrac{\cancel{3}(x+3)(3x-4)}{\cancel{3}}$

Nov 4-9:19 AM

Using the Common Factor Quotient (CFQ) Method of Factoring:

To factor $ax^2 + bx + c$:

1. The first step is to set up the following: $\dfrac{(ax\quad)(ax\quad)}{a}$
2. Find two factors of ac that have a sum of b.
   That is find e and f so that $ef = ac$, and $e + f = b$.
3. Place e and f as shown: $\dfrac{(ax+e)(ax+f)}{a}$
4. Check to see if there is anything that can be factored out of the parentheses in each of the 2 sets.
5. Reduce the fraction, if possible.

Example: Factor $24x^2 + 25x + 6$.

Nov 4-9:25 AM

Next, let's illustrate the Tic-Tac-Toe Method.

Factor the following trinomial.

b) $12x^2 - 11x - 36$

$12x^2$ (a)	-36 (b)	$-432x^2$ (c)
$3x$ (d)	-9 (e)	$-27x$ (f)
$4x$ (g)	4 (h)	$16x$ (i)

Target Sum: f + i

$-11x$

$(3x+4)(4x-9)$

-432

$12 \quad -36$

$3 \quad 4 \quad -9 \quad 4$

$-27 \quad 16$

$3x^2+5x-12$

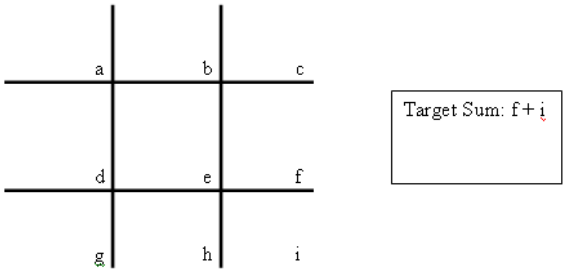
$\boxed{\text{Target Sum: } 5x}$

$3x^2$	-12	$-36x^2$
$3x$	3	$9x$
x	-4	$-4x$

$(3x-4)(x+3)$

Nov 5-8:43 AM

Using the Tic-Tac-Toe Method of Factoring



Example: Factor $24x^2 + 25x + 6$ (same example as CFQ).

Steps:

1. Put the first term of the trinomial in cell *a*

2. Put the last term of the trinomial in cell *b*

3. Cell *c* is equal to the product of cell *a* times cell *b* (first term times last term)

4. The target sum is equal to the middle term.

5. The next step is the key cells, *f* and *i*. Cell *f* times cell i should equal cell *c* and add to the target sum. At this point cell *f* and *i* are interchangeable

6. Cell *d* is the greatest common factor of cell *a* and cell *f.*

7. The rest of the cells can be filled in by finding the missing factor (cell *d* times cell *e* results in cell *f*; cell d times cell g results in cell *a*, …)

8. The factors of the trinomial are on the diagonals of cells d e
g h

Nov 4-9:28 AM

More practice: Factor completely.

a) $6x^2 - 22x - 8$

b) $12x^2 - 23x - 24$

c) $14x^2 - 31x + 12$

Nov 4-9:30 AM

Nov 5-9:11 AM