Sec. 7.7 (cont.) - Quadratic Equations and Applications

Additional HW: Sec. 7.7 #49, 54, 55, 59, 60, 62, 67

Agenda:
- Discuss what discriminant tells us.
- Solving quadratic word problems.
- Time to work / answer questions?

Reminder: Quiz tomorrow (7.4 - 7.7)
Also, PS4 due tomorrow.
Also, Exam 3 on Thursday.

Nov 10-8:25 AM

The expression $b^2 - 4ac$ is called the **discriminant**.

We can use the discriminant to determine the types of solutions we can expect.

Complete the table below:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

No real solutions.
One (repeated) *rational* solution.
Two distinct *rational* solutions.
Two distinct solutions

Discriminant	**Solutions**
$b^2 - 4ac > 0$	Two distinct solutions
$b^2 - 4ac > 0$ *and* a perfect square	Two distinct *rational* solutions.
$b^2 - 4ac = 0$	One (repeated) *rational* solution.
$b^2 - 4ac < 0$	No real solutions.

Nov 6-8:49 AM

Developing Understanding:

1. How can we use the quadratic formula to determine whether a quadratic equation is factorable?

(X) = 0

e.g. Is $4x + 3x^2 + 7$ factorable?

$b^2 - 4ac = 4^2 - 4(3)(7)$

$< 0 \Rightarrow$ no sol'ns

2. Given the quadratic equation $-2x^2 + bx + 8 = 0$. Is it possible to find b so that the quadratic equation has:

a) two rational solutions, b = 6

b) two real (non-rational) solutions, b = 1

c) one real solution, no way $b^2 + 64 = 0$

d) no real solutions? no way $b^2 + 64 < 0$ either.

$b^2 - 4(-2)(8)$

$= b^2 + 64$

$(6^2 + 64) = 100$ ✓

(10^2)

For each of the above, **find b (if possible)** OR explain **why it is not** possible.

Nov 6-9:03 AM

Developing Understanding:

How are solutions to the quadratic formula related to the ZPP?

(This should help you think about PS4 #3)

Gina solved $2x^2 + 5x - 3$ by factoring. She wrote:

$(2x - 1)(x+3) = 0$

$\Rightarrow$ $x = 1/2$ or $x = -3$

Trent solved it using the quadratic formula. He wrote:

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-3)}}{2(2)} = \frac{-5 \pm \sqrt{49}}{4}$$ ✗

$x = \frac{-5 \pm \sqrt{25 + 24}}{2(2)}$

$= \frac{-5 \pm \sqrt{49}}{4} = \frac{2}{4}$ or $\frac{-12}{4}$

$= \frac{1}{2}$ or -3

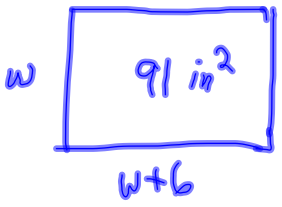
Is either student correct? Why?

Nov 10-8:53 AM

Area Word Problem

The length of a rectangle is 6 inches more than its width. The area of the rectangle is 91 square inches. Find the dimensions of the rectangle.

Let w = the width (in.)



$w(w+6) = 91$

$w^2 + 6w = 91$

$w^2 + 6w + 9 = 91 + 9$

$\sqrt{(w+3)^2} = \pm\sqrt{100}$

$w + 3 = \pm 10$

$w = -3 \pm 10$

So $w = 7$ or $w = -13$

$w = 7$, $l = 13$

Nov 10-8:46 AM

Two Numbers Word Problem

The product of two consecutive odd integers is 1 less than four times their sum. Find the two integers.

Let n = the 1st odd integer.

$(n)(n+2) = 4(n+n+2) - 1$

$n^2 + 2n = 8n + 7$

$n^2 - 6n - 7 = 0$

$(n-7)(n+1) = 0 \Rightarrow n = 7$ or $n = -1$

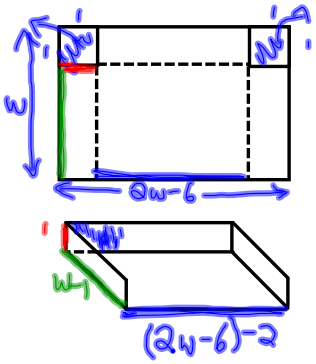
check: $7 \cdot 9 \stackrel{?}{=} 4(16) - 1$ ✓

check: $(-1)(1) \stackrel{?}{=} 4(0) - 1$ ✓

Nov 10-8:47 AM

Folding Box Application (c.f. PS4 #2)

A company is designing a steel letter tray. The machine that bends the steel works best with rectangular sheets of steel whose length is 6 inches less than twice the width. A 1x1 square will be punched two corners before the steel is bent up to form the tray (as shown). If the volume of the tray needs to be 108 cubic inches, what size steel sheet should the company start with?



$V = l \cdot w \cdot h$

$108 = (w-1)(2w-6-2(1))$

$108 = (w-1)(2w-8)$

$108 = 2w^2 - 10w + 8$

$-108 \quad -108$

$\frac{0}{2} = \frac{2w^2 - 10w - 100}{2} = \frac{2(w^2-5w-50)}{2}$

$0 = w^2 - 5w - 50$

$0 = (w-10)(w+5)$

So $w = 10$ or $w \neq -5$

Nov 10-9:21 AM

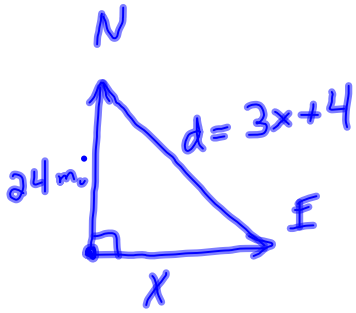
Distance and Pythagorean Theorem Application

Two cars leave an intersection. One car travels north; the other travels east. When the car traveling north had gone 24 miles, the distance between the cars was four miles more than three times the distance traveled by the car heading east. Find the distance between the cars at that time.

Pyth. Theorem:

$x^2 + 24^2 = (3x+4)^2$

HW: you solve it.



Nov 10-8:47 AM

Projectile Motion Application

An object is projected directly upward from the ground. After t seconds its distance in feet above the ground is $s = 144t - 16t^2$

A) After how many seconds will the object be 128 feet above the ground?

B) When does the object hit the ground?

$s = 144t - 16t^2$

A) $128 = 144t - 16t^2$

(consider factoring out 16)

$\div 16$

$8 = 9t - t^2$

$t^2 - 9t + 8 = 0$

$(t-8)(t-1) = 0$

$t = 8, \; t = 1$

B) $0 = 144t - 16t^2$

$0 = t(144 - 16t)$

so $t = 0$ or $t = \frac{-144}{-16} = 9$ sec.

Nov 10-8:47 AM

Nov 10-9:40 AM