Agenda:

1. Sec. 8.5 - Quadratics Final Notes
   - Deepening Understanding
   - An Application

2. Sec. 8.6 - Exponential and Logarithmic Functions
   Sec. 8.6 HW #1-4, 9, 15, 25, 26, 29-31, 37, 39, 53, 57

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Deepening Understanding:

$f(x) = a(x-h^2)+k$

1. What is the relationship between the x-intercepts and the solutions to the quadratic equation

$$\underbrace{0 = a(x-h)^2 + k}$$

solutions give x-intercepts.

the points on the graph of $y = f(x)$ where $y = 0$.

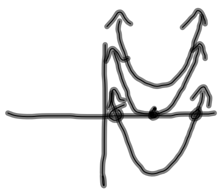
2. What would you conjecture about the number and type of x-intercepts for a *perfect square trinomial*?

$$\frac{0}{a} = \frac{a}{a}(x-h)^2 \cancel{+k}$$

$$0 = (x-h)^2$$

$$\pm\sqrt{0} = x-h$$

$$h = x \leftarrow \text{one solution (repeated root)}$$



Nov 24-9:46 AM

Deepening Understanding:

The quadratic formula was used to find that for a certain quadratic function:

$$x = \frac{-7 \pm \sqrt{81}}{2(-2)} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. What is the axis of symmetry for this quadratic function?

$x = \frac{-b}{2a} = \frac{-7}{2(-2)} = \frac{7}{4}$.

2. What are the x-intercepts? → $(-\frac{1}{2}, 0)$ and $(4, 0)$.

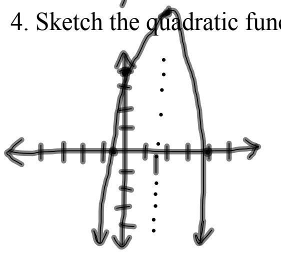
$\frac{-7+9}{-4} = -\frac{1}{2}$. $\qquad \frac{-7-9}{-4} = 4$.

3. What is the equation of the quadratic function?

$f(x) = ax^2 + bx + c$ $\qquad b^2 - 4ac = 81$. $\qquad 8c = 32$

$b = 7,\ a = -2$. $\qquad 49 - 4(-2)c = 81$ $\qquad c = 4$.

4. Sketch the quadratic function you found above. $f(x) = -2x^2 + 7x + 4$.



$f(\frac{7}{4}) = -2(\frac{7}{4})^2 + 7(\frac{7}{4}) + 4$

$= -2(\frac{49}{16}) + \frac{49}{4} + \frac{16}{4}$

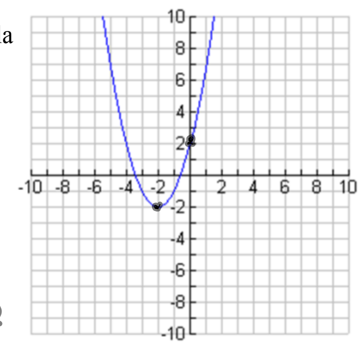
$= \frac{49}{-8} + \frac{98}{8} + \frac{32}{8}$

$= 81/8$

Nov 24-9:12 AM

Deepening Understanding:

Find the equation of the parabola plotted here.



$a > 0$

vertex: $(-2, -2)$

y-int: $(0, 2)$.

$f(x) = a(x+2)^2 + -2$

$f(0) = \underbrace{2 = a(0+2)^2 - 2}$

$2 = a(4) - 2$

$4 = 4a$

$1 = a$.

$f(x) = (x+2)^2 - 2$. $\quad (x+2)(x+2)$

$f(x) = x^2 + 4x + 4 - 2 = \boxed{x^2 + 4x + 2}$

? $f(-\frac{1}{2}) = 0$? $\quad (-\frac{1}{2})^2 + 4(-\frac{1}{2}) + 2 = \frac{1}{4} + \cancel{-2} + \cancel{2} = \frac{1}{4}$

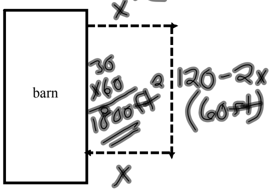
Nov 24-9:44 AM

An Application: *Optimization*

We often wish to find the largest or smallest value of some quantity. If the quantity can be modeled with a quadratic equation, we can use the vertex to find the desired value.

A farmer has 120 feet of fencing. He wants to put a fence around 3 sides of a rectangular plot of land, using his barn as the 4th side. Find the maximum area he can enclose. What dimensions give this area?

$x = (30\text{ft})$



$A = (\text{length})(\text{width})$

$A = x(120-2x) = (30)(60)$

$A = 120x - 2x^2$

$A = -2x^2 + 120x + 0$

opens downwards

$$\boxed{x = -\frac{b}{2a} = \frac{-120}{2(-2)} = 30}$$

$A = 1800\text{ft}^2$

Nov 25-8:44 AM

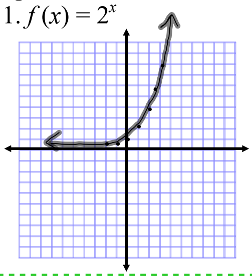
Section 8.6 - Exponential and Log Functions

Definition: An exponential function with base b where $b > 0$ and $b \neq 1$ is a function of the form

$f(x) = b^x$, where x is any real number.

Examples:

1. $f(x) = 2^x$

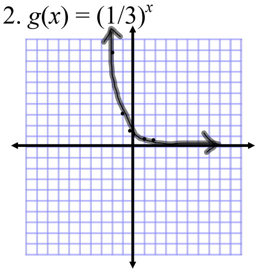


x	f(x)=2^x
-2	.25
-1	.5
0	1
1	2
2	4
2.5	2^2.5 = 5.6569...
3	8

$2^{-1} = \frac{1}{2^1}$

"x^y" "^"

2. $g(x) = (1/3)^x$



x	g(x)=$(1/3)^x$
-2	9
-1	3
-1/2	1.732...
0	1
1	$1/3 = .\overline{3}$
2	$1/9 = .\overline{1}$

$\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1$

$\left(\frac{1}{3}\right)\wedge\left(-\frac{1}{2}\right)$

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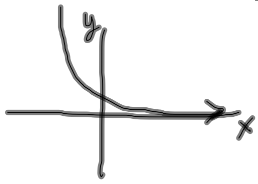
In general, the graph of the exponential function $f(x)=b^x$ has one of the following basic shapes:

1. If $b > 1$, it will exhibit *exponential growth*:

2. If $0 < b < 1$, it will exhibit *exponential decay*:



(Note: If $b < 0$, then b^x is not a well-defined function -- just try calculating $(-2)^{.5}$ on your calculator.)

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Exponential growth: A famous conundrum.

In this troubled economy, you seek employment as a lowly farm hand. The wily old farmer, who amassed a great fortune working in his previous job as a statistics consultant, agrees to hire you for exactly 30 days, but says,

"I'm feeling generous today, but I will only hire you under the following conditions: You will be paid 1 penny on the first day, 2 pennies on the second day, 4 pennies on the ~~fourth~~ day, 8 pennies on the ~~fifth~~ day, etc., and I will continue to double your daily wage until the end of the month. Do you accept my terms?"

You pause for a moment... should you take his offer? After a little bit of quick figuring, you look the farmer square in the eye and say, ... (what?)

n	P	C
1	1¢	1¢
2	2¢	3¢
3	4¢	7¢
4	8¢	15¢
5	16¢	31¢
6	32¢	63¢
⋮	⋮	
n	$2^{(n-1)}$	$(2^n)-1$
⋮		
30	2^{29}	$2^{30}-1$

1,073,741,824 − 1

$ 10,737,418.23

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Logarithms:

The inverse of addition is subtraction: (x + 4) - 4 = x.

The inverse of multiplication is division: (x*4) ÷ 4 = x.

The inverse of exponentiation is... the logarithm!

$\log_2(2^x) = x$ $\log_4(4^x) = x$

In general, we say that the "log base b of x" (written $\log_b x$) is the exponent that we must raise b to to give us x.

Examples:

A) $\log_5 25 = 2$ because... $5^2 = 25$

B) $\log_8 64 = 2$ because... $8^2 = 64$

C) $\log_4 64 = 3$ because... $4^3 = 64$ $4 \cdot 4 \cdot 4$

D) $(1/2)^3 = 1/8$. In logarithm form, this would say:

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$\boxed{\log_{(1/2)}\left(\frac{1}{8}\right) = 3}$

$\log_2\left(\frac{1}{8}\right) = (-3)$ $2^{(-3)} = \frac{1}{8}$

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To summarize:

"Logarithms are exponents."

(That is, the "log-base b" of a given number x is the exponent of b that would produce x. For instance, $\log_2 32 = 5$ because $2^5 = 32$.)

Exponential growth may start out slow but in the long run it produces very large growth.

(Exponential growth occurs whenever the base is greater than 1, as in $f(x) = 5^x$.

Exponential decay gives initially rapid decay that slows down over time.

(Exponential decay occurs whenever the base is between 0 and 1, as in $g(x) = (0.1)^x$.

Sec. 8.6 HW #1-4, 9, 15, 25, 26, 29-31, 37, 39, 53, 57

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