Mth126 - Agenda for 12/8/08

- Final Exam: Location & Day
    Saturday, 10-noon, in CWH 228 (capacity 56)
    Monday, 10-noon, in Cowley 041 (capacity 38)

I need a tentative head count for each day.

- Turn in projects (Part I)
    - Reminder: Part II due to D2L by 5pm today

- Handout Review for Final Exam

- Return Quiz 7, Discuss.

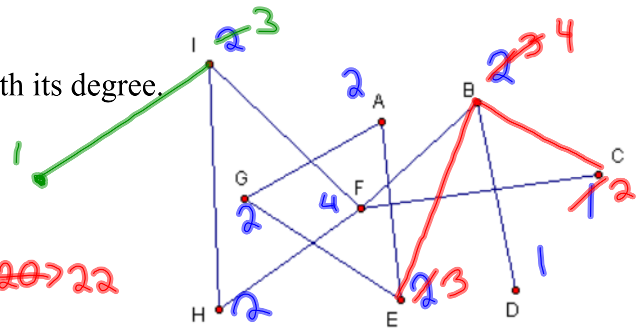
- Short quiz tomorrow - Sequences + Graph Theory

- Graph Theory, continued -- Euler Circuits
    HW #3-7 odd, 20-21, 23-29 odd, 33

Last Time

Definition: The **degree** of a vertex is the number of edges joined to that vertex.

Label each vertex with its degree.



~~18~~ → ~~20~~ → 22

What is the sum of the degrees? How is this related to the number of edges?

~~9~~ → ~~10~~ → 11

* Try adding edges to the graph above. Does the relationship between edges and vertices still hold?

* Create your own graph and see if this relationship holds.

(Theorem) 2(# edges) = sum of degrees

1

Last Time:

1. Read the definitions of **walk**, **path**, and **circuit** on pp. 857-58, and read Example 7 that follows.

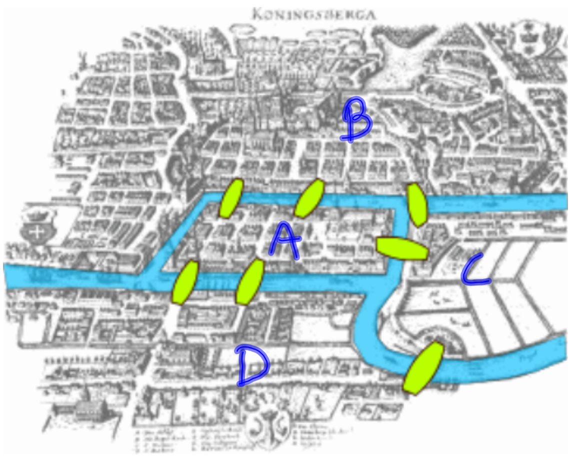
2. Read the definition of **complete graph** on p. 859, and read Example 9 that follows.

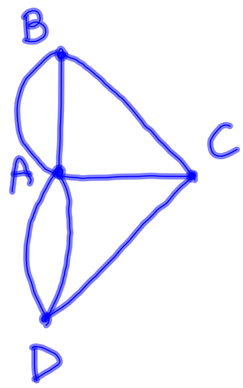
Walk: sequence of vertices linked by edges
Path: a walk that does not reuse edges
Circuit: a path that starts and stops at the same vertex.

Complete graph: one (and only one) edge between every pair of vertices.

15.2 - A famous problem: The 7 bridges of Konigsberg.

Can you cross each of the 7 bridges exactly once, returning to your point of origin?





(Leonard Euler ("oiler", 1707-1783) worked on this problem, among many others. Euler produced about 1000 books & papers, over half of them after he went blind at age 58).

Definitions:

An **Euler path** is a path that uses each edge of the graph exactly once.

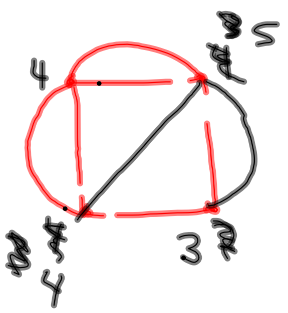
An **Euler circuit** is a circuit that uses each edge of the graph exactly once.

??? How is that different from a regular "path" or "circuit"?

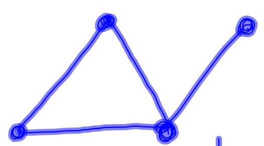
"Euler" means "use every edge"

Question: Under what circumstances does a graph have an Euler circuit?

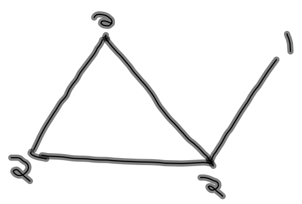
Let's try some examples & see what we can learn.



Note: there can be no



graph with a single odd degree vertex. (Odd vertices come in pairs).



Euler's Theorem:

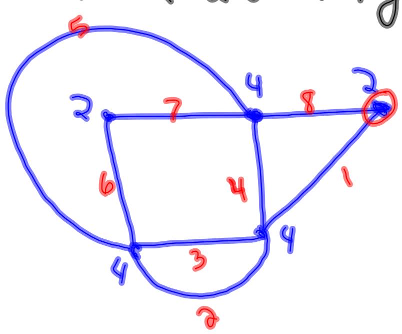
For a connected graph...

1. If a graph has an Euler circuit, then each vertex of the graph has even degree.
2. If each vertex of the graph has even degree, then the graph has an Euler circuit.

Create a graph that has an Euler circuit, and try to trace an Euler circuit (there can be more than one).

What goes wrong if there is a vertex with an odd degree?

must use at least 7 edges.



To prove that "if each vertex of a connected graph has an even degree, then the graph has an Euler circuit," we provide an algorithm for actually finding an Euler circuit in any such graph.

It's called Fleury's algorithm (p. 871):

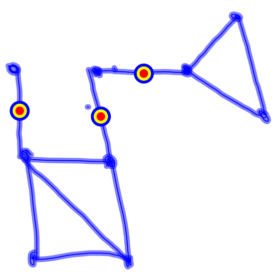
1. Start at any vertex. Go along any edge from this vertex to another vertex. Remove this edge from the graph.

2. You are now at a new vertex in a revised graph. Choose *any edge that is not a cut edge* (unless you have no other option). Go along the selected edge, and remove it from your graph.

3. Repeat step 2 until you have used all edges and gotten back to the vertex at which you started.

What is a cut edge? It is an edge which, if removed, disconnects a component of the graph.

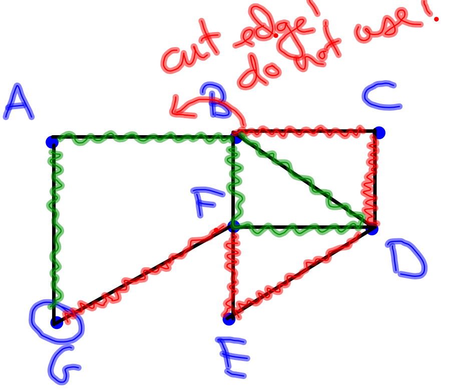
Create an example of a graph with one (or more) cut edges.

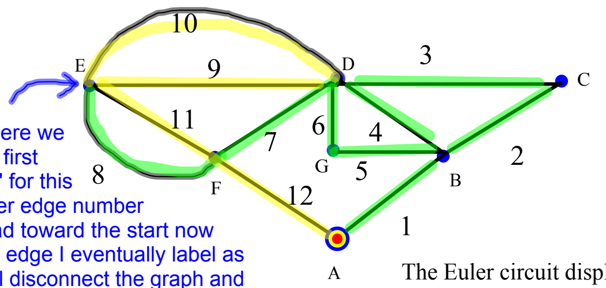


This graph has three different cut edges (marked with bullseyes)

Use Fleury's algorithm to find an Euler circuit in the following graphs.

G → F → E → D → C → B → F → D → B → A → G





Here's where we reach our first "cut edge" for this graph (after edge number 8). If I head toward the start now (along the edge I eventually label as "11"), I will disconnect the graph and not be able to cross all edges.

The Euler circuit displayed here is:
A B C D B G D F E D E F A.
(Many others are possible).