Mth171 - Exam 2 Name: KEY
Fall 2008

Note: The "Rules" for forming semi-regular tilings are listed on the separate sheet accompanying this exam.

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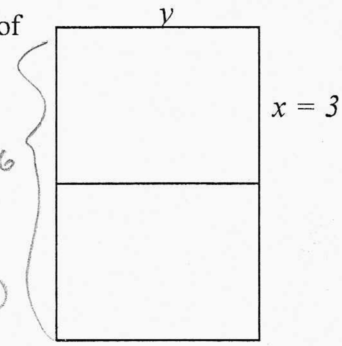
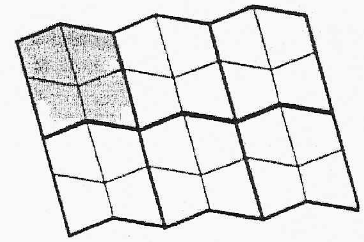
1. The rectangular tile shown below is a reptile so that when two copies of the rectangle are pieced together, the resulting figure is similar to the original rectangle. If x is 3 cm, what is y?



$$\frac{\text{long}}{\text{short}} = \underbrace{\frac{y}{3} = \frac{6}{y}}_{\text{solve for } y} \Rightarrow y^2 = 18$$

$$\Rightarrow y = \sqrt{18} = 3\sqrt{2} \text{ cm}$$

2. **True or False?** The tiling shown here (created from a collection of identical trapezoids) is a semiregular tessellation.



(Trapezoids are not regular polygons.)

3. **Explain:** What is a semiregular tessellation (or tiling)? What is the difference between a *semiregular* tessellation and a *regular* tessellation?

A semi regular tessellation involves more than one regular polygon it its construction, whereas a regular tessellation uses exactly one type of regular polygon. In either type of tessellation, each vertex configuration must be the same throughout the tiling.

4. Give a convincing argument why Rule 2 for semiregular tilings is correct.
Rule 2: Every semiregular tiling must have at least 3 polygons and no more than 6 polygons meeting at each vertex.
Be sure you address both parts. (Why at least 3? And why no more than 6?)

① If we used only 2 polygons, the vertex angles of one of them would need to be greater than or equal to 180° in measure – there is no such regular polygon.

② The regular polygon with the smallest vertex angles is the equilateral triangle (60°). Six of these together makes 360° and will surround a point. Thus, we cannot have more than six regular polygons surrounding a point or else the angles would exceed 360° in total measure!

You may use scratch paper to make sketches / tracings if it helps you answer the two questions on this page. Please be sure that any sketches you refer to are clearly labeled, and that your discussion is easy to follow.

5. Compare and contrast the four tilings associated with the following configurations. Which (if any) can be extended to semiregular tilings? Which (if any) are identical?

   Tip: All of these tilings satisfy Rule 1.

   A: 12.12.3
   B: 3.10.15
   C: 10.15.3
   D: 3.15.10

Of these configurations, only 3.12.12 (equals 12.12.3) extends to a semi-regular tiling.

If they extended to tilings, B and C would be identical to one another: the configuration 3.10.15.3.10.15, 3.10, 15 ... could simply be recorded as underlined, to express B as C.

6. Compare and contrast the four tilings associated with the following configurations. Which (if any) can be extended to semiregular tilings? Which (if any) are identical?

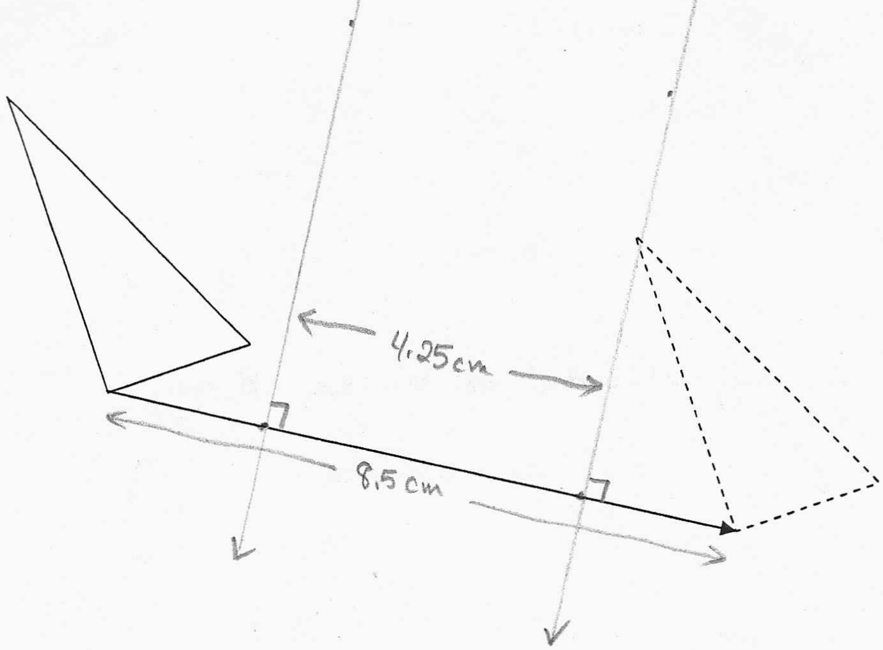
   Tip: All of these tilings satisfy rule 1.

   E: 3.4.4.6
   F: 4.6.3.4
   ~~G: 3.6.4.6~~ 3.4.6.4
   ~~H: 4.6.3.6~~ 6.4.3.4

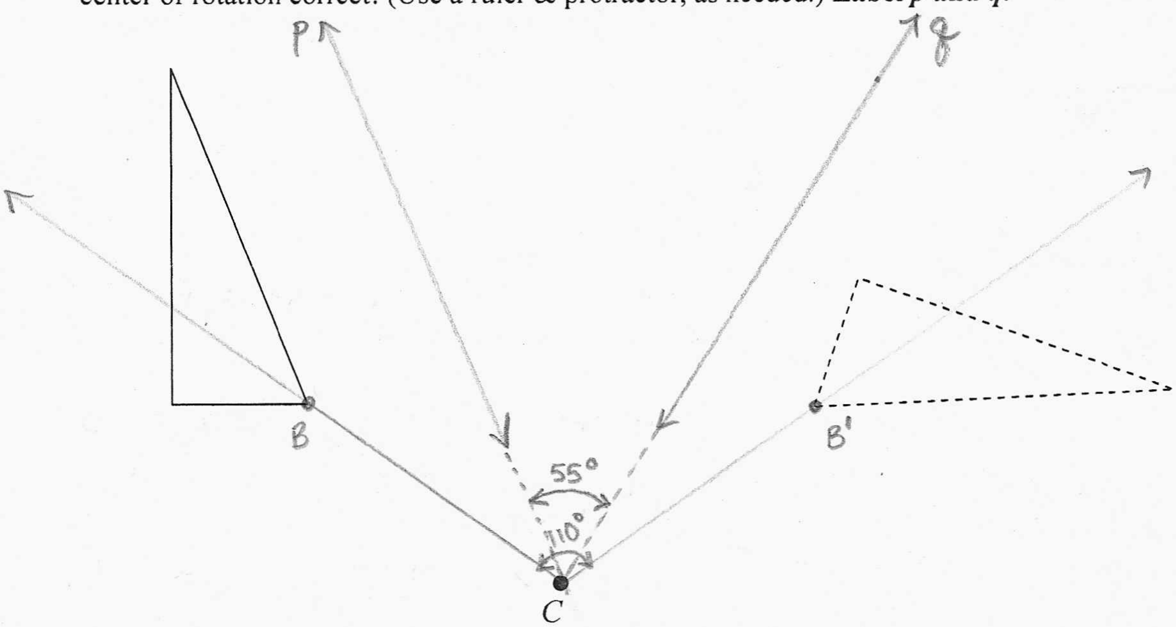
Tilings E and F are identical and will not extend to a semi-regular tiling due to Rule 5 (k.n.m.n is required if k is odd).

Likewise, tilings G and H are identical and since each satisfies Rule 5 we expect they will extend to a tiling. A careful sketch can be used to confirm this is true.

7. Draw two lines *l* and *m* so that the product of the reflection across line *l* followed by the reflection across line *m* will be the translation shown. Make sure you get the direction and distance correct! (Use a ruler and protractor, as needed). **Label *l* and *m*.**



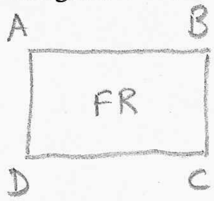
8. Draw two lines *p* and *q* so that the product of the reflection across line *p* followed by the reflection across line *q* will be the rotation about point *C* that is shown below. Make sure you get the angle and center of rotation correct! (Use a ruler & protractor, as needed.) **Label *p* and *q*.**



9. Consider the rectangle below. It has a vertical line of symmetry (call this left-to-right flip F) as well as a two-fold rotational symmetry (Rotate by 180°; call this rotation move R). Then its rosette group is $D_2 = \{1, R, F, RF\}$.

a) The result of the identity transformation "1" is shown above. Label the vertices of the other three rectangles to indicate the results of the transformations R, F, and RF.

b) The combination of a flip followed by a rotation is denoted FR. Sketch a picture of the configuration for FR. Which element of D_2 gives the same result as FR?



this shows FR = RF.

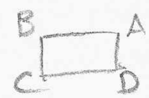
c) Let F_H denote a flip across the horizontal line of symmetry of the rectangle (in the direction perpendicular to flip *F*). Why didn't we need to include F_H in the rosette group D_2?

F_H would produce the same transformation as RF, and so it would be redundant to include it as an extra element of D_2.

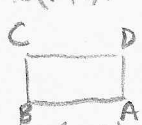
d) Below is the beginning of the multiplication table for D_2, the dihedral group consisting of the symmetries of a general rectangle. Complete the multiplication table. You may draw pictures if it helps you, but pictures are not required.

	1	**R**	**F**	**RF**
1	1	R	F	RF
R	R	1	RF	F
F	F	RF	1	R
RF	RF	F	R	1

$R^2F = 1F = F$

$F(RF) \to$  $= R$

$(RF)(RF) = R(FR)F = R(RF)F = R^2F^2 = 1$

$(RF)R =$  $= F$

$(RF)F = R(F^2) = R(1) = R$

Bonus question (optional): Is the multiplication table for D_2 commutative? Why or why not?

Yes! For each pair of transformations α and β in D_2, the multiplication table confirms that $\alpha\beta = \beta\alpha$. (The symmetry across the main diagonal confirms this.)