Agenda:

- Wrap up Star Polys
- Revisit Gauss's Theorem
- Review

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Star Polygons: {8,k}

N	k	regular?	num. of revolutions	# of cycles	Length of each cycle
8	1	Yes	1	1	8
8	2	No	2=1x2	2	4
8	3	Yes	3	1	8
8	4	No	4=1x4	4	2
8	5	Yes	5	1	8
8	6	No	6=3x2	2	4
8	7	Yes	7	1	8

K

cycle x length = N

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Star Polygons: {9,k}

N	k	regular?	num. of revolutions	# of cycles	Length of each cycle
9	1	Yes	1	1	9
9	2	Yes	2	1	9
9	3	No	3=1x3	3	3
9	4	Yes	4	1	9
9	5	Yes	5	1	9
9	6	No	6=2x3	3	3
9	7	Yes	7	1	9
9	8	Yes	8	1	9

1
5
9
4

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Star Polygons: {12,k}

N	k	regular?	num. of revolutions	# of cycles	Length of each cycle
12	1	Yes	1	1	12
12	2	No	2=1x2	2	6
12	3	No	3=1x3	3	4
12	4	No	4=1x4	4	3
12	5	Yes	5	1	12
12	6	No	6=1x6	6	2
12	7	Yes	7	1	12
12	8	No	8=2x4	4	3
12	9	No	9=3x3	3	8
12	10	No	10=5x2	2	6
12	11	Yes	11	1	12

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Star Polygons: {13,k} if gcf ≠ 1 then compound (and vice-versa)

N	k	regular ?	num. of revolutions =k	# of cycles gcf	Length of each cycle n÷gcf
15	1	Yes	1	1	15
15	2	Yes	2	1	15
15	3	No	3=1x3	3	5
15	4	Yes	4	1	15
15	5	No	5=1x5	5	3
15	6	No	6=2x3	3	5
15	7	Yes	7	1	15
15	8	Yes	8	1	15
15	9	No	9=3x3	3	5
15	10	No	10=2x5	5	3
15	11	Yes	11	1	15
15	12	No	12=4x3	3	5
15	13	Yes	13	1	15
15	14	Yes	14	1	15

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12. * Rule for when a star polygon is regular?

    * Rule for when it's compound?
        - How many cycles will it have?
        - How long is each cycle?

13. Rule for how many revolutions around the center?

14. Number of cycles in a {24,9}? How long? *How could you check this quickly?*

15. Which {21,k} polygons are regular?

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Gauss's Theorem:

Which regular polygons can be constructed using straightedge and compass alone?

So far, we know we can construct...
- a "regular" triangle (equilateral),
- a "regular" quadrilateral (square)
- a regular pentagon.

What *else* can we construct?

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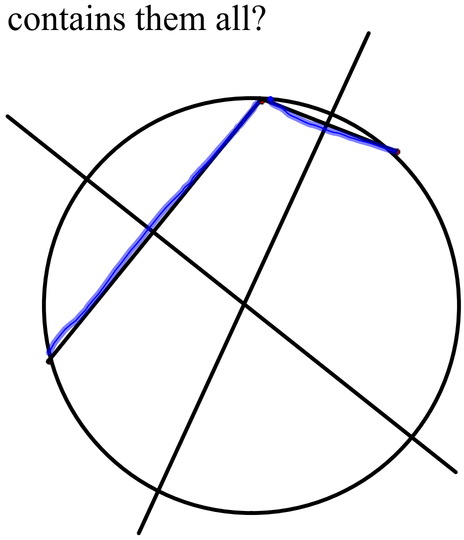
First, an intermediate construction:

Given 3 arbitrary points, how could you construct a single circle that contains them all?



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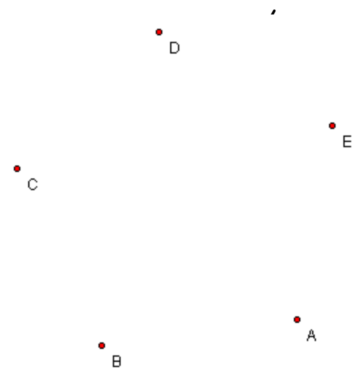
Here's a regular 5-gon. Construct a circle through its vertices.



Steps:
1. Construct perpendicular bisectors for two of the sides.
2. These two lines will intersect at the center of the desired circle.
3. Form a circle with radius extending from the center to any of the five original points.

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Gauss's Theorem: A regular n-sided polygon can be constructed iff all odd prime factors of *n* are distinct Fermat primes (those of the form $2^{2^k} + 1$).

$k=3: 2^{2^{(3)}}+1$

List the Fermat numbers for k = 0, 1, 2, 3, 4, 5.

2^8+1

257

The first five Fermat primes are:

F0 = 3 (prime)
F1 = 5 (prime)
F2 = 17 (prime)
F3 = 257 (prime)
F4 = 65537 (prime)
~~F5 = 4294967297 (prime?)~~ = 641 * 6700417: NOT prime.
(This shows that not all numbers of the form $2^{2^k} + 1$ are prime! Those that *are* prime are known as the Fermat primes.)

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Gauss's Theorem: A regular n-sided polygon can be constructed iff all odd prime factors of *n* are distinct Fermat primes (those of the form $2^{2^k} + 1$).

F0 = 3 (prime)
F1 = 5 (prime)
F2 = 17 (prime)
F3 = 257 (prime)
F4 = 65537 (prime)

Quick Exercises: According to Gauss, is it possible to construct…

1. …a regular 15-sided polygon using ruler and compass? ✓ $3 \cdot 5$
2. …a regular 60-sided polygon? ✓ $15 \times 4 = 3 \cdot 5 \cdot 2^2$
3. ~~3.~~ ✗ … a regular 45-sided polygon? $3 \cdot 3 \cdot 5$
4. ~~4.~~ ✗ …a regular 42-sided polygon? $6 \cdot 7$

Additional Review:

Potential exam topics include:

- Paper Billiards
  * Where should you aim to make a bank shot?
    A double bank shot?

- Straightedge & Compass Constructions
  * Also, reasoning about constructions... why do they work?

- Reasoning about the Golden Ratio
  * Exercise: If AB/AC = AC/CB = phi, why is phi = 1/phi + 1?
  * Exercise: How can you derive phi = (1+sqrt5)/2 from the equation phi = 1/phi + 1?

- Theoretical Origami (and Mira)
  * Know how to do the basic constructions (using Mira)
    (up through and including copying an angle)
    Exercise: Try copying an angle with a Mira.
    Exercise: What other constructions can you perform using a Mira?

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