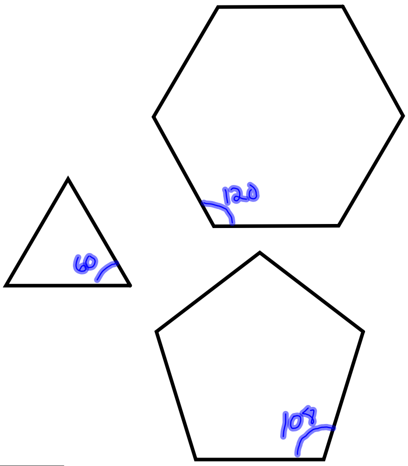
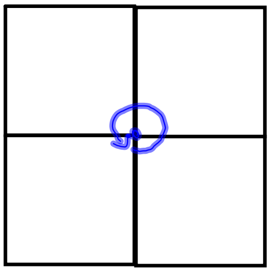
4.1 - Regular and Semi-regular Tilings

A *regular* tiling consists of repeated copies of a single regular polygon, meeting edge to edge so that every vertex has the same configuration.



1. Which regular polygons can tile the plane?

(there are only three of these... how can you be sure?)



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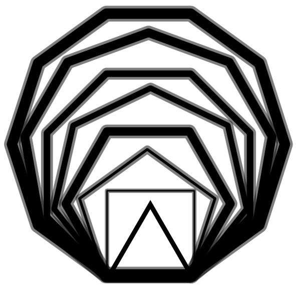
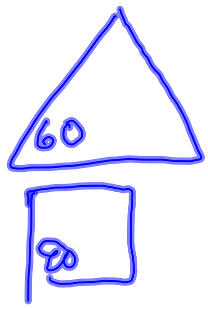
A *semi-regular* tiling consists of repeated copies of more than one type of regular polygon so that every vertex has the same configuration.



(there are exactly 8 of these... our task today is to use reasoning to find all 8.)



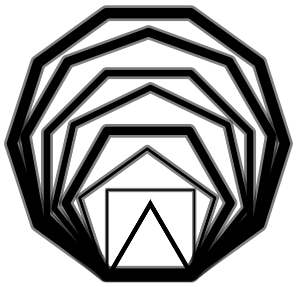
3. What is the largest number of regular polygons that can fit around a vertex?

6

Rule 2: There must be at least 3 and no more than 6 polygons meeting at each vertex.

Tesselation Shapes

4. Can there be four different polygons at a vertex? (What would the angle measures be?)



△ - 60
□ - 90
n=5 ⬠ - 108
n=6 ⬡ - 120

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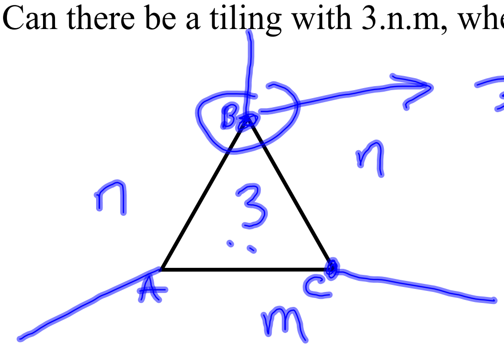
Rule 3: No semiregular tiling can have four different types of polygons meeting at a vertex.

Tesselation Shapes

Can there be a tiling with 3.n.m, where n is different from m?



→ 3.n.n

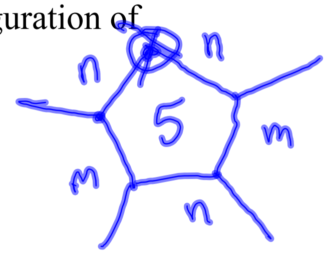
So n=m!

6. Show that one cannot have a vertex configuration of 5.n.m where n is different from m.

What about 6.n.m? → n.m.6

7.n.m?

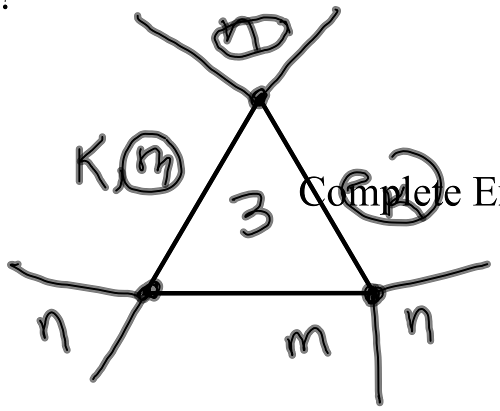
k.n.m?



Rule 4: No semiregular tiling can have vertex configuration k.n.m where k is odd and n is not equal to m.

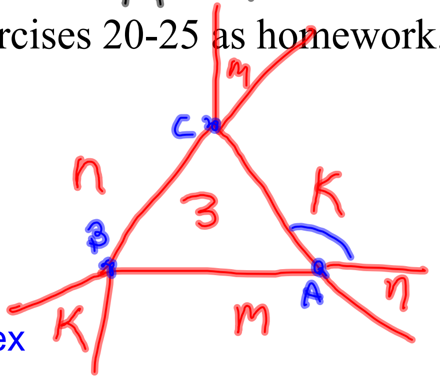
~~7.6.3.4~~

Try the configuration 3.k.n.m. What restrictions can you find?



K=m

Complete Exercises 20-25 as homework.



Rule 5: No semiregular tiling can have vertex configuration 3.k.n.m unless k=m.

A: 3K.n.m
B: 3mKn
C: 3nmK

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Vertex Configurations for Possible Tilings (p. 88)

Symbol:	# Polygons	Comment		
	6	#3		
3.3.3.3.6	5	#7	(lots of triangles)	
3.3.3.4.4	5	#7		
4.4.4.4	4	#1		
4.4.3.6 = 3.6.4.4 ⊗ try 3.4.6.4	4	#8		A
6.6.3.3 → 3.6.3.6 → ~~3.3.6.6~~	4	#8	(remember rule 3)	
3.3.4.12	4	#8		
6.6.6	3	#1		
4.8.8	3	from text		
3.7.42	3	#9		
12.12.3	3	#9a		
5.5.10 ↔ 10.5.5	3	#9a	(contain duplicate polygons)	B
4.5.20	3	#9b		
4.6.12	3	#9b	(contain a square)	C
3.9.18	3	#9c		
3.8.24	3	#9c	(contain one equilateral triangle)	D
3.10.15	3	#9c		

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10. Draw a section of the three regular tilings.

11. Rule out the tilings with three polygons that do not work (use Rule 4).
(note: 6.6.6, 4.8.8, work,
but 3.7,42 does not).

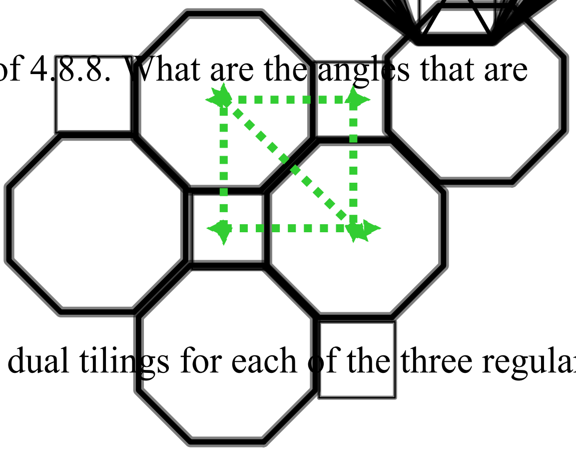
12. Draw a section of tiling 3.4.6.4.

Do #13 and 14 after class.

Dual Tilings

A dual tiling is formed from a regular or semiregular tiling by connecting the adjacent *centers of gravity*.

#18. Sketch the dual tiling of 4.8.8. What are the angles that are formed by the dual?



#19. Draw and describe the dual tilings for each of the three regular tilings.

Complete Exercises 20-25 as homework.

Summary of key points raised in class:

Rule 4 says that if k is odd, then no tiling of the form k.n.m is possible unless n=m.

Rule 5 says that 3.k.n.m is not possible unless k = m.

Notice that a.b.c.d = b.c.d.a = c.d.a.b = d.a.b.c, which amount to just starting your counting at a different vertex and going around.

However, a.d.c.b would be different! That's because in that case, d and b do not share a side whereas in a.b.c.d they DO share a side.

E.g. 3.6.4.4 is: triangle-hexagon-square-square-(triangle-hexagon...)
and 3.4.6.4 is: triangle-square-hexagon-square-(triangle-square...)

> Now, Rule 5 says that 3.6.4.4 won't work (even though it can also be denoted as 4.3.6.4), but we can fix it by rearranging the shapes. We could do 3.4.6.4 instead (which can also be denoted as 6.4.3.4, or numerous other ways if you like) .

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