Agenda:

- Topics for Exam 2
- Wrap-up Sec. 5.1 (Reflections)
  * Key principles of reflected images
  * Task: Result of two reflections across intersecting lines
- Rosette Groups and Point Symmetry

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<u>Topics for Exam 2</u>

* Sec. 4.1 - Tesselations, Regular, and Semi-regular Tilings
  (I'll provide the "rules" for semi-regular tilings, but may ask you to justify one or more of them)

* Sec. 4.2 - Irregular Tilings
  (Escher style transformations)

* Sec. 5.1 - Kaleidoscopes and Reflections
* Sec. 5.2 - Rosette Groups and Rotational Symmetry

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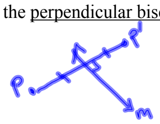
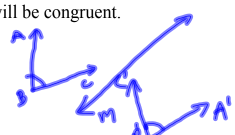
Key principles of reflections:

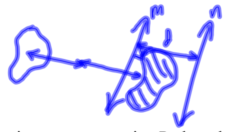
1. When a point P is reflected across a line *m* to create the image P', line *m* will be the perpendicular bisector of segment PP'.




2. When an angle ABC is reflected across a line *m* to create the image A'B'C', the angles will be congruent.

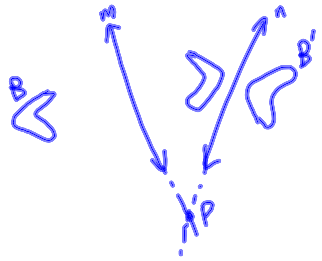



3. If *m* and *n* are parallel lines, then the composition $R_n$ o $R_m$ is equivalent to a translation in the direction perpendicular to *m* towards *n*. The length of the translation is equal to twice the distance from *m* to *n*.




4. If *m* and *n* are lines that intersect at a point *P*, then the composition $R_n$ o $R_m$ is equivalent to a ...

...rotation about point *P* through twice the angle from *m* to *n*

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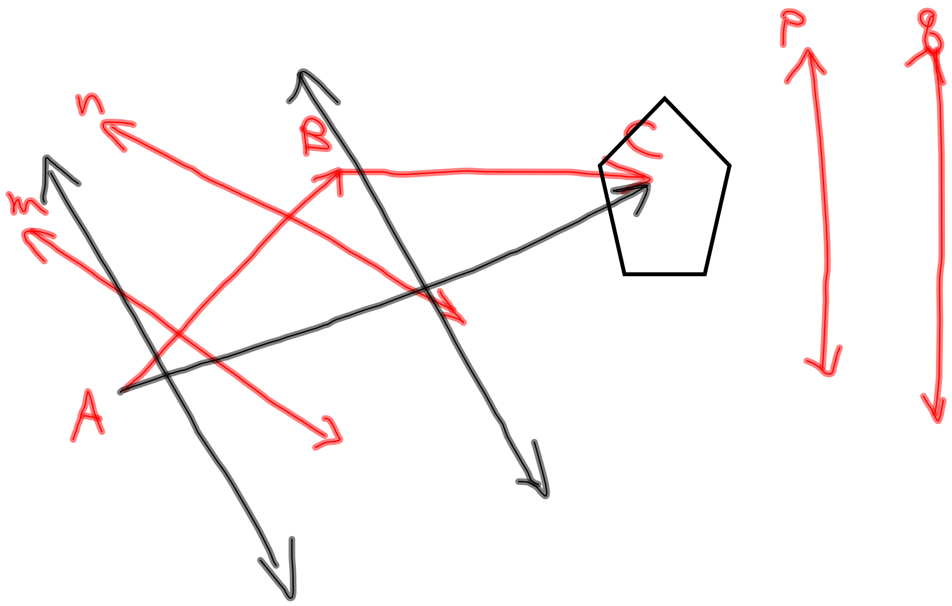
#5) Slide me now

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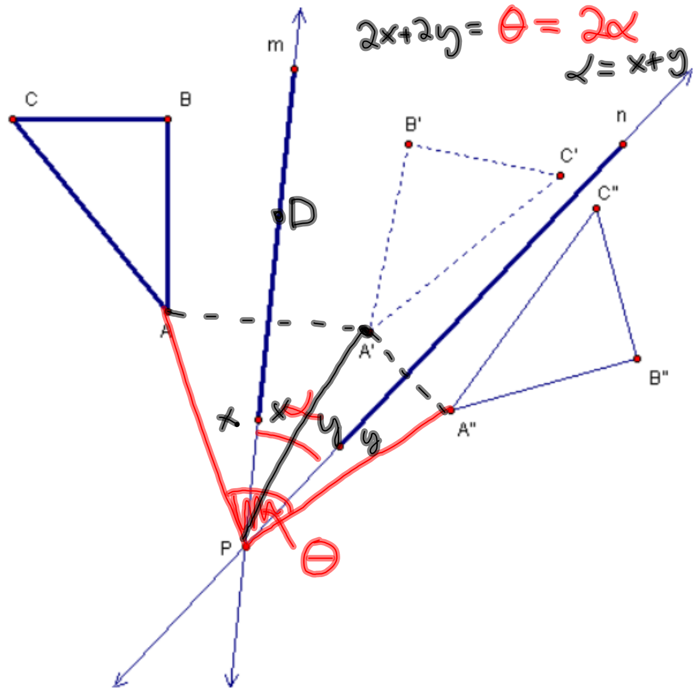
4. If $m$ and $n$ are lines that intersect at a point $P$, then the composition $R_m$ o $R_n$ is equivalent to a ...

...rotation about point $P$ through twice the angle from $m$ to $n$

Proof idea:

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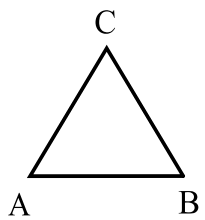
Sec. 5.2 - Rosette Groups



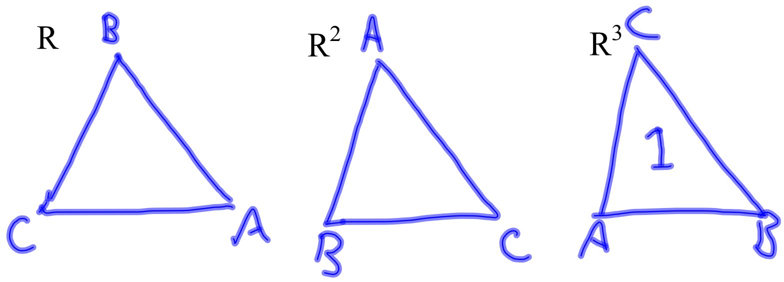

Starting with an equilateral triangle, we can rotate counterclockwise by 120° to get an identical triangle.

We can denote this action as $R_{120}$, a counter clockwise rotation of 120 degrees.

We'll be working with the triangle for a while, so for simplicity let's agree that R means a 120 degree counterclockwise rotation of a triangle. Likewise, R°R, or simply $R^2$ represents two subsequent rotations.

Sketch the result of:

$R^3 = 1$

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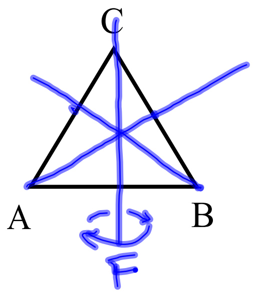


Our triangle also has three lines of reflection. (sketch these)

Let us denote reflection across the vertical line by $F$ (for *flip*).

Then $F^2 = 1$ (the identity).

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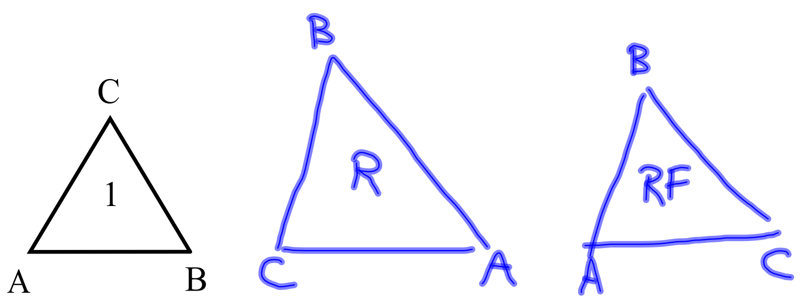
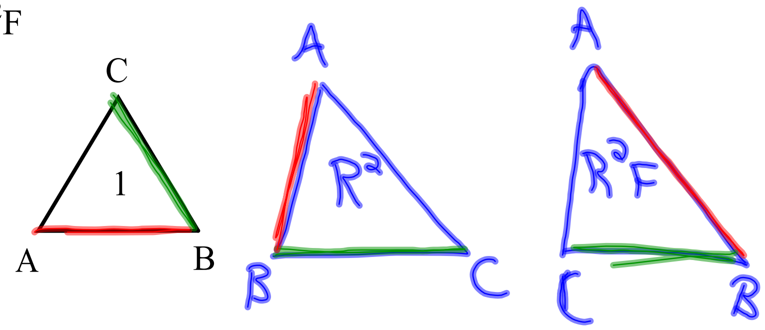
We might also combine flips and rotations and observe how these act on the triangle.

Sketch the result of applying the following transformations:

RF




$R^2F$

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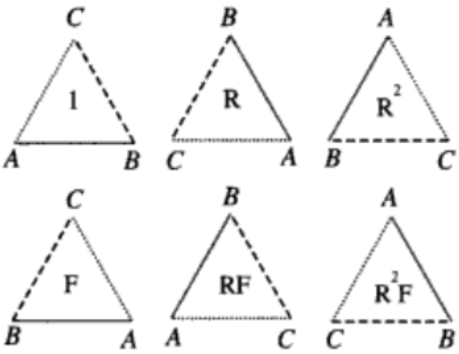
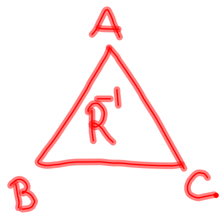
To summarize, we have found the following six configurations, or re-arrangements, of the triangle:




$R^{-1} = R^2$

Are there other configurations?

2. Try reflecting the original triangle across the line from A through BC. Which configuration does this give?

3. What if you reflect the original across the line from B to AC?

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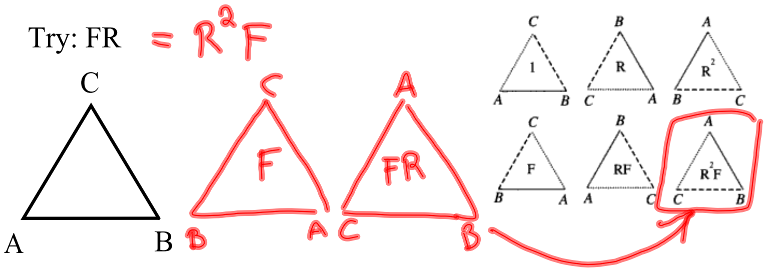
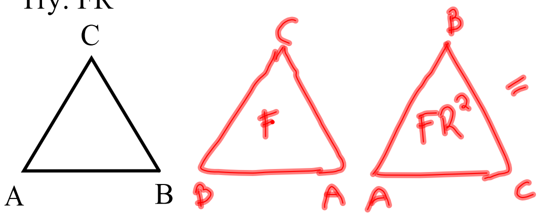
We have listed rotations first, then flips. Would we obtain anything new by doing flips first, and then rotations? Other combinations?

Try: FR $= R^2F$

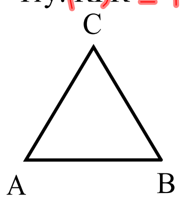



Try: FR$^2$




$= RF$

Try: (RF)R $= F$

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The list of unique operations on the triangle is denoted $D_3$, called the *dihedral group of a 3-fold rotation*. It is also called the *rosette group* of the triangle.

We can combine operations to form a sort of "multiplication table".

**$D_3$: Symmetries of an Equilateral Triangle**

| | 1 | $R$ | $R^2$ | $F$ | $RF$ | $R^2F$ |
|---|---|---|---|---|---|---|
| 1 | 1 | $R$ | $R^2$ | $F$ | $RF$ | $R^2F$ |
| $R$ | $R$ | $R^2$ | 1 | $RF$ | $R^2F$ | $F$ |
| $R^2$ | $R^2$ | 1 | $R$ | $R^2F$ | $F$ | $RF$ |
| $F$ | $F$ | $R^2F$ | $RF$ | 1 | $R^2$ | $R$ |
| $RF$ | $RF$ | $F$ | $R^2F$ | $R$ | 1 | $R^2$ |
| $R^2F$ | $R^2F$ | $RF$ | $F$ | $R^2$ | $R$ | 1 |

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Note: "Products" in the dihedral group obey an associative property. ✓

Do they obey a commutative property? no

$R^{-1}$ $(R^{-1})(R) = 1$

**$D_3$: Symmetries of an Equilateral Triangle**

| | 1 | $R$ | $R^2$ | $F$ | $RF$ | $R^2F$ |
|---|---|---|---|---|---|---|
| 1 | 1 | $R$ | $R^2$ | $F$ | $RF$ | $R^2F$ |
| $R$ | $R$ | $R^2$ | 1 | $RF$ | $R^2F$ | $F$ |
| $R^2$ | $R^2$ | 1 | $R$ | $R^2F$ | $F$ | $RF$ |
| $F$ | $F$ | $R^2F$ | $RF$ | 1 | $R^2$ | $R$ |
| $RF$ | $RF$ | $F$ | $R^2F$ | $R$ | 1 | $R^2$ |
| $R^2F$ | $R^2F$ | $RF$ | $F$ | $R^2$ | $R$ | 1 |

Compute:

$(FRF) = R^2$

$(RF)R^2 = R^2F$

$(FR^2F)R = (R)R = R^2$

$\boxed{R^{-1} = R^2}$

$R^{-2} = R$

$F^{-1} = F$

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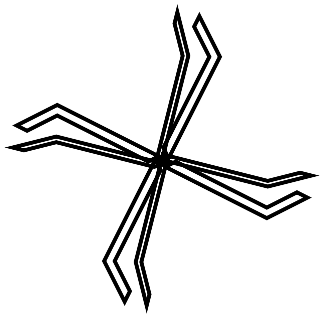
The dihedral group is composed of flips and rotations.

Another type of rosette group is the cyclic group $C_n$.
It consists of only the $n$-fold rotations but no reflections.



The figure above has no reflectional symmetries, but it has rotational symmetry. The cyclic group $C_4$ describes its rosette group.

$C_4 =$

Write the multiplication table for $C_4$.

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Homework for next time:

#5-10, 19, 20

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