

Agenda:

0. Review Taxicab HW
1. Class Activity - Spherical Geometry
2. Discussion of Spherical Geometry

HW: Big Ideas (p. 93) #2-7

Taxicab Geometry HW

- Solutions are online
- Questions?

<http://www.uwlax.edu/faculty/hasenbank/mth171/index.html>

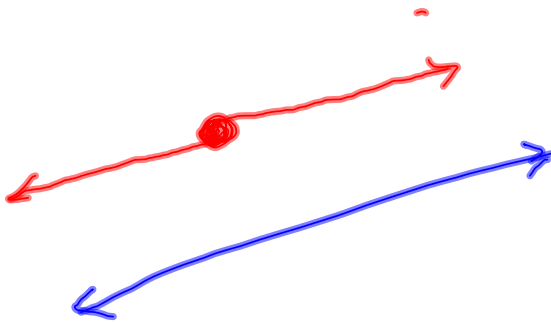
Question 7: Is there an equilateral triangle? (GSP)

11.13_TaxicabRegularTriangle.gsp

"Euclidean" Geometry is the geometry we get by assuming Euclid's five postulates and logically deriving additional results.

1. A unique line can be drawn from any point to any other point.
2. A line segment can be extended to produce a line.
3. A circle may be described with any center and distance.
4. All right angles are equal to one another.
5. Through a given point not on a line there can be drawn exactly one line parallel to the given line.

Our "Taxicab" geometry is a simple example of a non-Euclidean geometry (it fails to satisfy Euclid's 5th postulate).



Class Activity 14: Life on a Spherical World

1. Define a "straight line" on a sphere.
2. How long is a "straight line"? Find a parallel line. Is this geometry *Euclidean*?
3. Line segments -- How many are there?
4. Triangles -- Measure the angles.
-- Triangle angle sums.
5. Triangles -- Right triangles. Does Pythagorean Theorem hold?
6. Perpendicular Lines: How many are there for a given line through a given point?

Discussion:

Recall: For a sphere with radius r , the surface area is: $A = 4\pi r^2$.

Implication: There is an upper bound on the size of geometric objects that can be drawn on a sphere.

Therefore:

- * There is a largest circle (find one).
- * There is a longest line segment (find one).
- * There is a largest triangle (find one).



Euclid's 5th Postulate:

There is a unique parallel line through any point not on a given line. *As we saw, this is false in spherical geometry.*

Implication:

There are no parallelograms! (Explain).

Can you find a square?
(First, what *is* a square?)



quadrilateral
4 ~~90~~ congruent angles ✓
4 congruent sides ✓

Implications:

1. There are two line segments between any two points.
(A major segment and a minor segment).



2. In Euclidean geometry, we say "a triangle is three non-collinear points and the line segments that join them."

In spherical geometry, we refine the definition to say that a triangle is 3 points and the *minor* line segments joining them. The interior is taken to be the smaller of the two areas enclosed by those segments.

Do we need to specify that the points be non-collinear?

Triangle angle sums:

A triangle with three 90 degree angles exists.

What is the smallest triangle angle sum you found?
the largest?

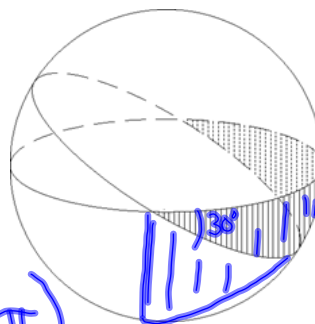
A two-sided polygon:

Since every pair of lines intersects in two antipodal points, we have a two sided polygon (with 2 vertices). It's called a *lune*.

1. What is the area of a lune with a 30° angle? (Take a unit sphere).

Hint: what if it was a 90° angle?

$$S = 4\pi r^2 \Rightarrow S = 4\pi \left(\frac{1}{4}(4\pi) = \pi.\right)$$



2. What will be the area of a lune if the angle is a radians?

http://www.uwosh.edu/faculty_staff/szydlis/elliptic/elliptic.htm



* Note: a full revolution is 2π radians.

Next time:

- Triangle areas (in terms of *lunes*)
- Hyperbolic geometry

11.13_TaxicabRegularTriangle.gsp