Spherical Geometry - Wrap-up

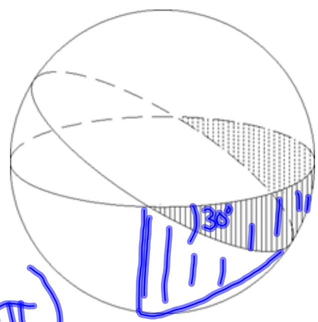
A) Use lunes to show the area of a triangle is (a + b + c) - pi.

B) (#6-7) Triangle congruence theorems: which "work" in spherical?

SSS	SAS	ASS
ASA	AAS	AAA

Spherical Geometry: A polygon with only two sides!

Since every pair of lines intersects in two antipodal points, we have a two sided polygon (with 2 vertices). It's called a *lune* (or a *biangle*).



1. What is the area of a lune with a 30^{o} angle? (Take a unit sphere).

Hint: what if it was a 90° angle?

$S = 4\pi r^2 \Rightarrow S = 4\pi \quad \left(\frac{1}{4}\right)(4\pi) = \pi.$

2. What will be the area of a lune if the angle is *a* radians?

$\frac{\pi}{2} \rightarrow A = \frac{1}{4}(4\pi) = \pi$

$\frac{\pi}{6} \rightarrow A = \frac{1}{12}(4\pi) = \frac{\pi}{3}$

$a \text{ radian} \rightarrow A = \frac{a}{2\pi}(4\pi) = 2a$

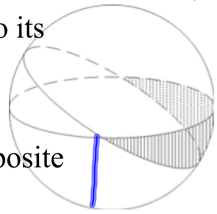
* Note: a full revolution is 2*pi* radians.

A) Derive area of triangle using lunes.

http://www.uwosh.edu/faculty_staff/szydliks/elliptic/elliptic.htm

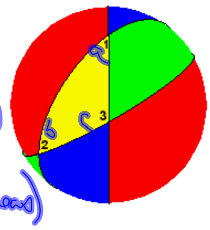
<u>Theorem:</u> The area of a lune with (radian) angle a is $2a$.

<u>Definition:</u> The *spherical excess* of a triangle with angles a, b, and c (measured in radians) is defined to be $\boxed{a+b+c - \pi.}$



<u>Conjecture:</u> The area of a spherical triangle is equal to its spherical excess.

<u>Proof:</u> Any triangle defines 3 overlapping lunes, plus 3 "twin" lunes (and a "twin" triangle) on the opposite side of the sphere.



So... $2(2a) + 2(2b) + 2(2c)$

$= (\text{surf area sphere}) + 2(\Delta\text{ area}) + 2(\Delta\text{ area})$

$4a + 4b + 4c = 4\pi(1^2) + 4(\Delta\text{ area})$

$a + b + c - \pi = \Delta\text{ area}$, as claimed.

$3\frac{\pi}{2} - \pi = \frac{\pi}{2}$

B) (#6-7) Triangle congruence theorems: which "work" in spherical?

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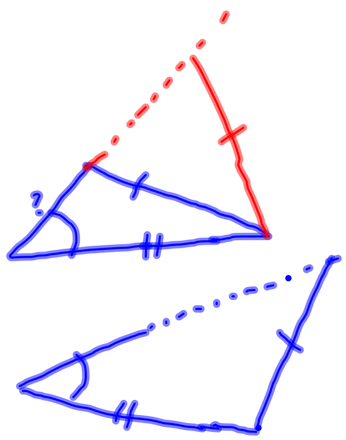
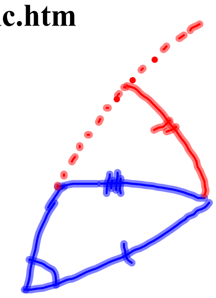


ASS



A final non-Euclidean geometry: Hyperbolic geometry

Euclid's Postulates:

1. A unique line can be drawn from any point to any other point.
2. A line segment can be extended to produce a line.
3. A circle may be described with any center and distance.
4. All right angles are equal to one another.
5. Through a given point not on a line there can be drawn ~~exactly~~ one line parallel to the given line.

at least 2

Let's see how Hyperbolic Geometry differs.
Do Class Activity 15: "Life on a Hyperbolic World"

Class Activity 15: Life on a Hyperbolic World

0. Create a model of the hyperbolic plane (use heptagons).

1. What will a 'straight' line look like on the hyperbolic plane?
   Are they finite or infinite in length?

2. Can you draw parallel lines? What do you notice about
   the distance between non-intersecting lines?

3. Given a line and a point not on the line, how many different
   parallel lines through the given point are there?

(Leave 4 & 5 for another time).

Hyperbolic geometry replaces Euclid's "parallel postulate" with the following:

The hyperbolic parallel postulate:

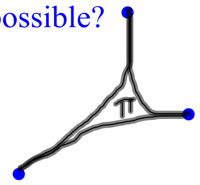
Given a line and any point not on the line, there are at least two lines through that point parallel to the given line.

An entirely new geometry resulted from that change. In this geometry...

* Triangle angle sums are always *less* than 180°.
* The area of a hyperbolic triangle (with angles a, b, and c) is $A = pi - (a + b + c)$.

Questions:

1. Describe what happens to the measures of the angles of an *equiangular* hyperbolic triangle as the area of the triangle increases. Can you sketch an example to show how this is possible?



2. What if the geometry of our universe is actually not Euclidean, but hyperbolic (or perhaps spherical)? Devise an experiment that would allow us to decide.

Why study non-Euclidean geometries?

1. To know what something is, it helps to know what it isn't.
2. There is increasing evidence that our universe may not actually be Euclidean at all... a fact that is very important in physics (esp. astronomy).

Homework:

p. 101 of *Big Ideas* handout: #2, 5, 6.

Also be working on your portfolio entries!