Mth171 - Agenda for 12/2/08

- Course evals.
- Return quizzes, discuss.

- Wrap up non-Euclidean discussion.
  * Polygon angle sums
  * Hyperbolic train tracks?
  * Regular pentagon with 90^{o} angles?

- An introduction to Topology

- Time for review and portfolio work
  (Reminder: Exam 3 on Thursday)
  (Reminder: Portfolios due next Tuesday)

Wrap up of Non-Euclidean:

To know what something is, it helps to know what it isn't.

Spherical triangles (on a unit sphere):

Triangle angle sums are greater than 180^{o} (or *pi* radians)

Area = (sum of angles) - *pi*

Hyperbolic triangles:

Triangle angle sums are less than 180^{o} (or *pi* radians)

Area = *pi* - (sum of angles)

2. Do similar but not congruent triangles exist?

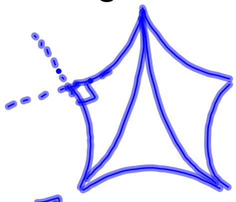
What about the AAA congruency theorem?

Hyperbolic railroad tracks

5. Can we build a set of railroad tracks on a hyperbolic plane? Support your answer.

What can you say about the sum of the angles of a regular hyperbolic pentagon?

$(5-2)180 = 540$



< 540

6. Can a right-angled regular pentagon exist on the hyperbolic plane? Support your answer.

$5 \times 90 = 450^\circ$

Topology

Topology is sometimes called "rubber sheet" geometry, because one essentially pretends that everything is made of extremely flexible rubber.

Topological equivalence:

Two objects are equal *topologically* if one can be deformed continuously into the other.

(Two objects are equal *geometrically* if they are congruent.)

A continuous deformation does not cut, tear, or glue pieces together.

Alphabet Topology: Classify the following letters into topologically equivalent groups.

B X DO K H

Q

P

AR

T Y E F

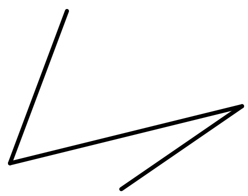
S Z L J I G

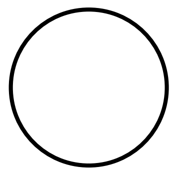
C M N V W U

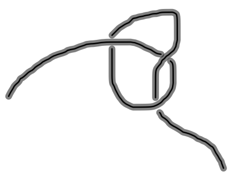
Topological dimension:

A point is 0 dimensional
A line is 1 dimensional
A plane is 2 dimensional
"Space" is 3 dimensional

Classify the following as 0, 1, 2, or 3 dimensional.







Covering dimension

Do Exercises 4-8 (p. 416):

4. Draw a finite collection of points. Cover with pennies with the fewest possible overlaps. What is the thickest overlapping of pennies you have? 1

5. Lay out a section of string that does not cross over itself. What is the thickest overlap of pennies? 2

6. Cut out (or just sketch) any shape from a flat sheet of paper. What is the thickest overlap of pennies? 3

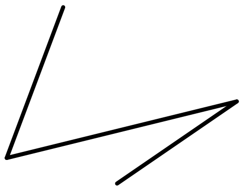
7. Think about a three dimensional object. (Now you need to use small spherical balls instead of pennies). 4

8. Conjecture a definition of *covering dimension*.

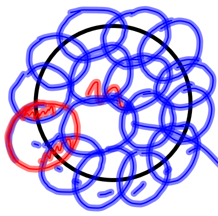
c.d = (Thickest overlap) − 1

Re-visit the following questions, using your definition of covering dimension.

What is the covering dimension of the following?

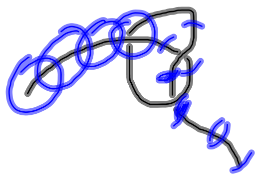


2-1=1



(boundary) 2-1=1

disc 3-1=2

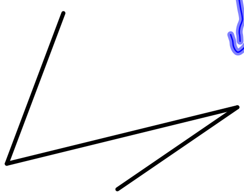


2-1=1

Embedding dimension:

The embedding dimension is the smallest dimensional space an object could be squeezed into (via continuous deformation).

What is the embedding dimension of the following?



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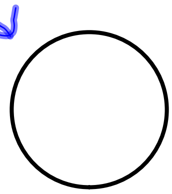
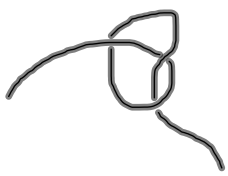
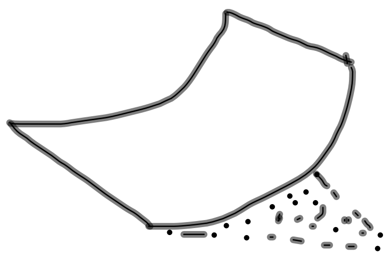


2



1



11. Find an object with covering dimension 1 that cannot be fit back into one- or two-dimensional space. (So it will have an embedding dimension of 3).

12. A point has 0 dimensions. Find a zero-dimensional object that requires one-dimensional space to exist.

14. Find an object with covering dimension 2 that cannot be fit back into 2-dimensional space.

Embedding dimension:

A) Take a long strip of paper and form a cylinder by holding the ends together.

How many "sides" does the cylinder have?
How many edges?
What does any small "neighborhood" look like?

B) Take your cylinder apart at the seam, give one end two twists, and put the ends back together.

What has changed?
What has stayed the same?

C) Take your modified cylinder apart, give one end only one twist, and put the ends back together.

What has changed?
What has not?
Note: You have created the infamous Mobius band.