Present Tuesday: 11.3 #13, 16, 25

9:55: Kaitlin D Amber E Alyssa F Molly G Kaitlin H Kaitlyn H
11:00: Jennifer K Taylor K Elizabeth L Kamry L Caitlyn M Genevieve M

HW from last night: Sec. 11.3 #5-15, 16, 24, 25

HW tonight: Sec. 11.3 #17-23, 26-31, 32, 34, 35-53 odd, 57-60.

Present Wednesday: 11.3 #23, 34, 51

9:55: Amanda J Grace J Molly K Azjja K Katie K Megan M
11:00: Danielle M Steve M Emily N Jordan P Thomas P Rebecca P

Remember to set your name placards on your desk :)

Mth126 Quick Quiz
Tue 02/01/2011

Name: ____________________
Section: 9:55 11:00

Counting numbers are to be formed using digits from the set {3, 4, 5, 6, 7}. Determine the number of ways that a four-digit number can be formed from this set if there is one pair of adjacent 5s and no other repeated digits. For example, one possibility is 3554 but 3553 would not be permitted.

choice:	place the "55"	place the 2 remaining digits	Total
# ways:	55 _ _, _ 55 _, _ _ 55 (3 ways)	(4×3) ways (no repeats)	3×4×3 = 36 ways

Scale:
3 = correct answer and work showing complete understanding.
2.5 = insignificant mistake, but work is mostly correct.
2 = mistake near end or could not finish (or relied on calc.)
1.5 = made significant mistake(s), but based on relevant principles.
1 = false start, but sustained effort based on relevant principles.
0.5 = false start, based on non-relevant principles.
0 = no progress at all.

Section 11.3 - Combinations and Permutations

Warm-up:

1. How many ways are there to elect a president and secretary of a math club with 5 members {A, B, C, D, E}?

$5 \cdot 4 = 20$ ways
$P(5,2)$

2. How many 2-person algebra committees can be created from the math club?

$C(5,2) = 10$

(half as many as above)

Permutations: How many different *arrangements* of 2 items can be selected from a group 5? (No repetitions, but *order matters*)

secr.

pres.	**A**	**B**	**C**	**D**	**E**
A	.	AB	AC	AD	AE
B	BA	.	BC	BD	BE
C	CA	CB	.	CD	CE
D	DA	DB	DC	.	DE
E	EA	EB	EC	ED	.

→ 5 x 4 = 20 ways

Combinations: How many different *subsets* of 2 items can be selected from a group of 5? (No repetitions, but *order doesn't matter*).

secr.

pres.	**A**	**B**	**C**	**D**	**E**
A	.	AB	AC	AD	AE
B	~~BA~~	.	BC	BD	BE
C	~~CA~~	~~CB~~	.	CD	CE
D	~~DA~~	~~DB~~	~~DC~~	.	DE
E	~~EA~~	~~EB~~	~~EC~~	~~ED~~	.

→ $\frac{5 \times 4}{2}$ = 10 ways

In general, the number of arrangements (or **permutations**) of *n* distinct things taken *r* at a time, can be calculated as

P(n,r) = (n)(n-1)(n-2)...(n-r+1)

$P(8,6) = 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3$

For instance:

A) P(5,2) = $5\cdot 4 = 20$

("How many ways are there to choose a pres. & secretary for the math club?")

B) P(8,3) = $8\cdot 7\cdot 6$

("How many three letter codes can be formed from the letters {A, B, C, D, E, F, G, H} without* repeating any letters?")

*What if repeats are allowed?

$\cancel{P(8,3)} \neq 8\cdot 8\cdot 8 = 8^3$

C) P(5,5) = $5\cdot 4\cdot 3\cdot 2\cdot 1 = 5!$

("How many ways can you arrange the members of the math club for a photograph?")

A more compact version of this formula can be found using factorial notation:

$$P(n, r) = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Verify the formula above gives the same answer as our previous formula:

A) P(5,2) = $\frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{\cancel{3\cdot 2\cdot 1}} = 5\cdot 4$

B) P(8,3) = $\frac{8!}{5!} = 8\cdot 7\cdot 6$

C) P(5,5) = $\frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5!$

* Note: 0! is defined to be equal to 1

Remember: Permutations are useful to count arrangements whenever order matters *and* repetitions are not allowed.

Combining Permutations and the FCP:

Together with the fundamental counting principle, the use of P(n,r) can save time by automating some steps in the counting process.

3. How many ~~10~~ 8-digit codes can be created if each must start with 2 letters followed by 4 digits and then 3 more letters, where repetitions are not allowed *within* any of the three groups of symbols?

$\underbrace{P(26,2)}_{\text{letters}} * \underbrace{P(10,4)}_{\text{digits}} * \underbrace{P(26,3)}_{\text{letters}}$

you finish:

eg: 26 nPr 2 = $26\cdot 25$

Combinations are used for counting *subsets*, meaning repetitions are not allowed and *order doesn't matter*. We write C(n,r) to refer to the number of *r*-item subsets that can be formed from a set of *n* items.

Consider: How many 3 person committees can be selected from the Math Club (with members {A,B,C,D,E}): $C(5,3)$

First let's list the possible committees; let's try to be systematic:

Starts with A: {ABC} {ABD} {ABE} {ACD} {ACE} {ADE}

Starts with B: {BCD} {BCE} {BDE}

Starts with C: {CDE}

Question: How many ways can the set {ABC} be rearranged (if order matters)? What about {ABD}? etc.?

$P(5,3) = 5\cdot 4\cdot 3 = 60$

$C(5,3) = \frac{P(5,3)}{3!} = \frac{60}{6} = 10$ ✓

In general, $\boxed{C(n,r) = \frac{P(n,r)}{r!}}$

Facts:

1. We have seen that $P(n,r) = \frac{n!}{(n-r)!}$.
2. **Permutations** count rearrangements **separately**.
3. **Combinations** treat all rearrangements as **the same set**.
4. Any set of *r* items has *r*! possible rearrangements.

Use those facts to explain why a formula for C(n,r) is:

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

Show that C(5,3) gives the correct number of 3-person committees that can be formed from the math club's 5 members.

Permutations: How many different *arrangements* of 2 items can be selected from a group 5? (No repetitions, but *order matters*)

secr.

pres.	**A**	**B**	**C**	**D**	**E**
A	.	AB	AC	AD	AE
B	BA	.	BC	BD	BE
C	CA	CB	.	CD	CE
D	DA	DB	DC	.	DE
E	EA	EB	EC	ED	.

→ 5 x 4 = 20 ways

Combinations: How many different *subsets* of 2 items can be selected from a group of 5? (No repetitions, but *order doesn't matter*).

secr.

pres.	**A**	**B**	**C**	**D**	**E**
A	.	AB	AC	AD	AE
B	BA	.	BC	BD	BE
C	CA	CB	.	CD	CE
D	DA	DB	DC	.	DE
E	EA	EB	EC	ED	.

→ $\frac{5 \times 4}{2}$ = 10 ways

Careful: It's not just that "half of them" are repeats -- but that there are 2! rearrangements of this 2-item set.

Combination or Permutation?

a) How many different five letter arrangements are there using the letters in mathisfun?

Permutation

$P(9,5) = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15{,}120$

b) Suppose you had a nine person basketball team, how many ways can you select a five-person starting line up?

Combination

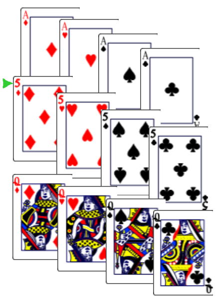
$c(9,5) = \frac{9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 9 \cdot 2 \cdot 7 = 126$

Poker Hands (see p. 654 for examples):

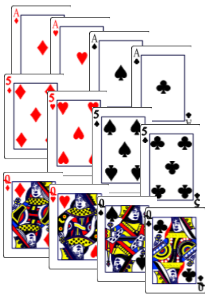
A "standard 52-card deck" has 52 cards consisting of:

13 different "face values" (Ace, 2, 3, 4, ... 9, 10, J, Q, K)
4 different "suits" (**Diamond**, **Heart**, **Club**, **Spade**)
- Diamonds and Hearts are **red**
- Clubs and Spades are **black**

So there are four of each face value, making 52 cards in the deck.



Poker Hands (see p. 654 for examples):



Practice Questions:

1. How many ways can you select a set of three Aces from the deck?

2. How many ways are there to select a pair of 5's from the deck?

3. How many ways are there to select any three cards of the same face value from the deck? (e.g. three 5s or three Ks, etc.)