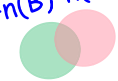
Present Wednesday: 11.3 #23, 34, 51

9:55: Amanda J Grace J Molly K Azjja K Katie K Megan M

11:00: Danielle M Steve M Emily N Jordan P Thomas P Rebecca P

Today's Objective: Learn to apply the Complements Principle and the Additive Counting Principle to solve counting problems involving Not and Or.

$n(A)+n(B)-n(A\cap B)$



Homework for Tonight:

Sec. 11.5 #1, 2, 4-6, 8, 15, 16, 21, 25-28, 37, 38

Present Thursday - 11.5 #26, 37, 38

9:55: Katie M Alyssa N Nicole N Rachel R Hannah R Chelsea S

11:00: Emma R Jamie S Erin S Jamie S Victoria S Abbey S

Also on the agenda:

* Return quizzes

* Quiz Thursday: Sec. 11.3 (nPr and nCr)

"flush" hands

① $52 \cdot 12 \cdot 11 \cdot 10 \cdot 9$ — what would this represent?

$= 52 \cdot P(12,4) = 4 \cdot 13 \cdot \cancel{(12 \cdot 11 \cdot 10 \cdot 9)/5!}$

② $P(13,5) = 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 / 5!$

This is missing a factor of 4, which accounts for the 4 different suits.

③ $4 \cdot C(13,5) = \left(\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5!}\right)(4)$

"Jeff Hubbards Home"

A ~~B~~ C ~~D~~ E F

choose 1st standard site	" 2nd standard site
6	5

2!

$= \frac{30}{2!} = \frac{P(6,2)}{2!} = C(6,2)$

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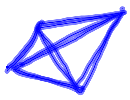
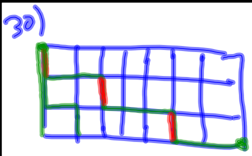
45)



$C(4,3) = \frac{4 \cdot 3 \cdot 2}{3!} = 4$

$C(20,3) =$

30)



$\underset{1^{st}}{D}\ \underset{2^{nd}}{D}\ \_\ D\ \ldots\ \underset{10^{th}}{\_}$

$C(10,3) = \frac{10 \cdot 9 \cdot 8}{3!}$

Notes about nPr and nCR (see HW #57-60 in Sec. 11.3)

57) C(12,9) = C(12, 3)

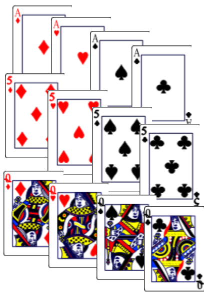
58) C(n, r) = C(n, n-r)

59) P(n, 0) = 1

60) C(n, 0) = 1

Do these properties make sense in the context of real world problems?

Poker Hands (see p. 654 for examples):

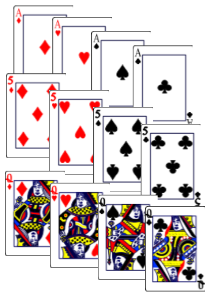


Practice Questions:

1. How many ways can you select a set of three Aces from the deck?

2. How many ways are there to select a pair of 5's from the deck?

Poker Hands (see p. 654 for examples):

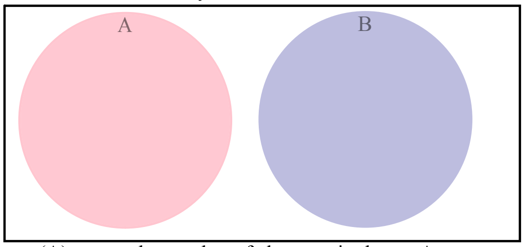


Practice Questions:

3. How many ways are there to select any three cards of the same face value from the deck? (e.g. three 5s or three Ks, etc.)

Sec. 11.5 - Counting with "Not" and "Or"

First, a little set theory review:



$n(A)$ means the number of elements in the set A.
$n(A')$ means the number of elements *not* in A.

$n(A \cup B)$ means the number of elements in the *union*.
$n(A \cap B)$ means the number of elements in the *intersection*.

Motivating Example:

1. How many proper subsets are there of the set {w, x, y, z}?
   (recall that a proper subset must not equal the set itself)

(at least two ways to proceed...)

0-items: $\{\ \} \rightarrow 1$ way $= C(4,0)$

1-item: $\{w\}, \{x\}, \{y\}, \{z\} \rightarrow 4$ ways $= C(4,1)$

2-items: $C(4,2) = \frac{4 \cdot 3}{2!} = 6$ ways

$\{w,x\} = \{x,w\}$

3-items: $C(4,3) = \frac{4 \cdot 3 \cdot 2}{3!} = 4$ ways

15 ways

The 'Complements Principle' of Counting

If A is any set within the universal set U, then:
$n(A) = n(U) - n(A')$

When phrases like *'at least one'* are used in a problem, it is often easier to apply the complements principle.

Example: How many different groups of three people could be chosen from the math club if at least one of them must be a freshman?

Math Club = {Amber, ~~Ben~~, ~~Chia~~, Danny, Edward, Fiona, & Zeb}
Freshmen = {Ben, Chia}

n(3 person groups altogether) $\quad C(7,3)$

− n(3 members, none of which are freshmen) $\quad - C(5,3)$

n(3 person groups w/ at least one freshman) = answer!

$\frac{7 \cdot \cancel{6} \cdot 5}{\cancel{3!}} - \frac{5 \cdot 4 \cdot \cancel{3}}{\cancel{3!}}$

$35 - 10 = 25$ ☺

Example: How many 5-card poker hands contain at least one heart?

n(poker hands total) → $C(52,5)$

− n(poker hands with no hearts) → $- C(39,5)$

n(poker hands with at least one heart)

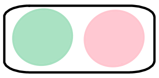
$\left(\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}\right) - \left(\frac{39 \cdot 38 \cdot 37 \cdot 36 \cdot 35}{5!}\right)$

$= 2{,}023{,}203$ ways

$C(13,1) \cdot C(39,4)$
$+ C(13,2) \cdot C(39,3)$
$+ C(13,3) \cdot C(39,2)$
$+ C(13,4) \cdot C(39,1)$
$+ C(13,5) \cdot C(39,0)$

The Special Additive Counting Principle

If A and B are disjoint sets, then
$n(A \cup B) = n(A) + n(B)$

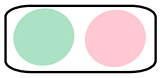


Example: How many two digit numbers are either odd or are multiples of ten?

$n(\text{2 digit odds}) + n(\text{2 digit mults of 10})$
$= (9 \times 5) + 9$
$= 45 + 9 = 54$

The Special Additive Counting Principle

If A and B are disjoint sets, then
$n(A \cup B) = n(A) + n(B)$

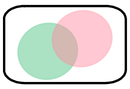


Example: How many five card hands consist of either all clubs or all red cards?

Example: If you toss six coins, in how many ways can you obtain at least two tails? (Note that since the coins are indistinguishable, order does not matter here).

The General Additive Counting Principle

If A and B are any two sets, then
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Example: How many two digit numbers are odd or multiples of 5?

odd: 45 | mult of 5: $2 \times 9 = 18$ | mult of 5 and odd: 9

$45 + 18 - 9 = 54$

Example: A single card is drawn from a deck. In how many ways could it be a heart or a face card?

Feedback time...

So, you are using these pre-class notes... excellent!
You'll notice I've been printing them 4 slides to a page to save space -- let me know if that's helpful or if you'd rather I print them full size (or somewhere in between).

Send me an email or just talk with be briefly before / after class someday. Happy note-taking!